

# Web Data Models

**XPath: Evaluation**

**Silviu Maniu**



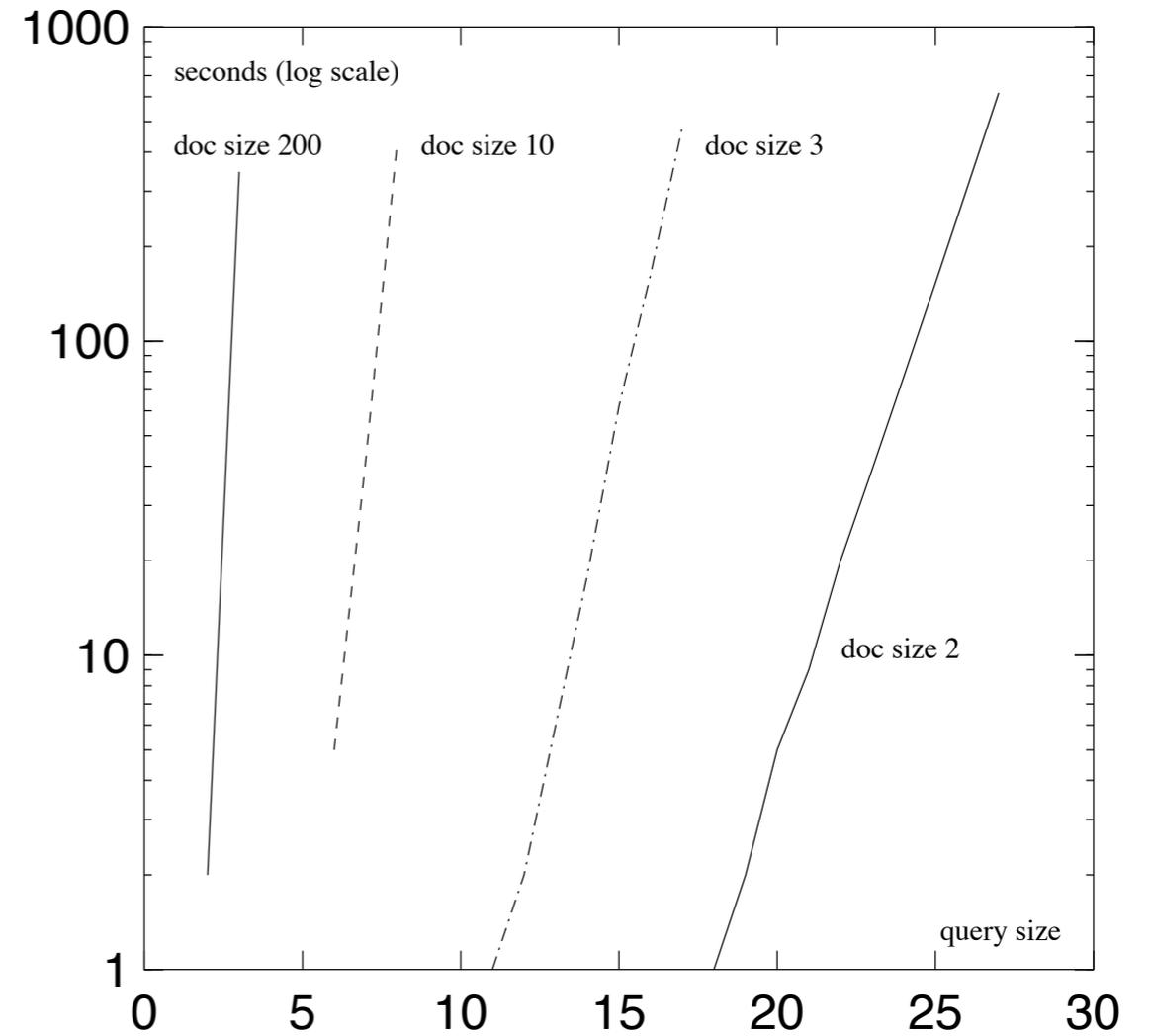
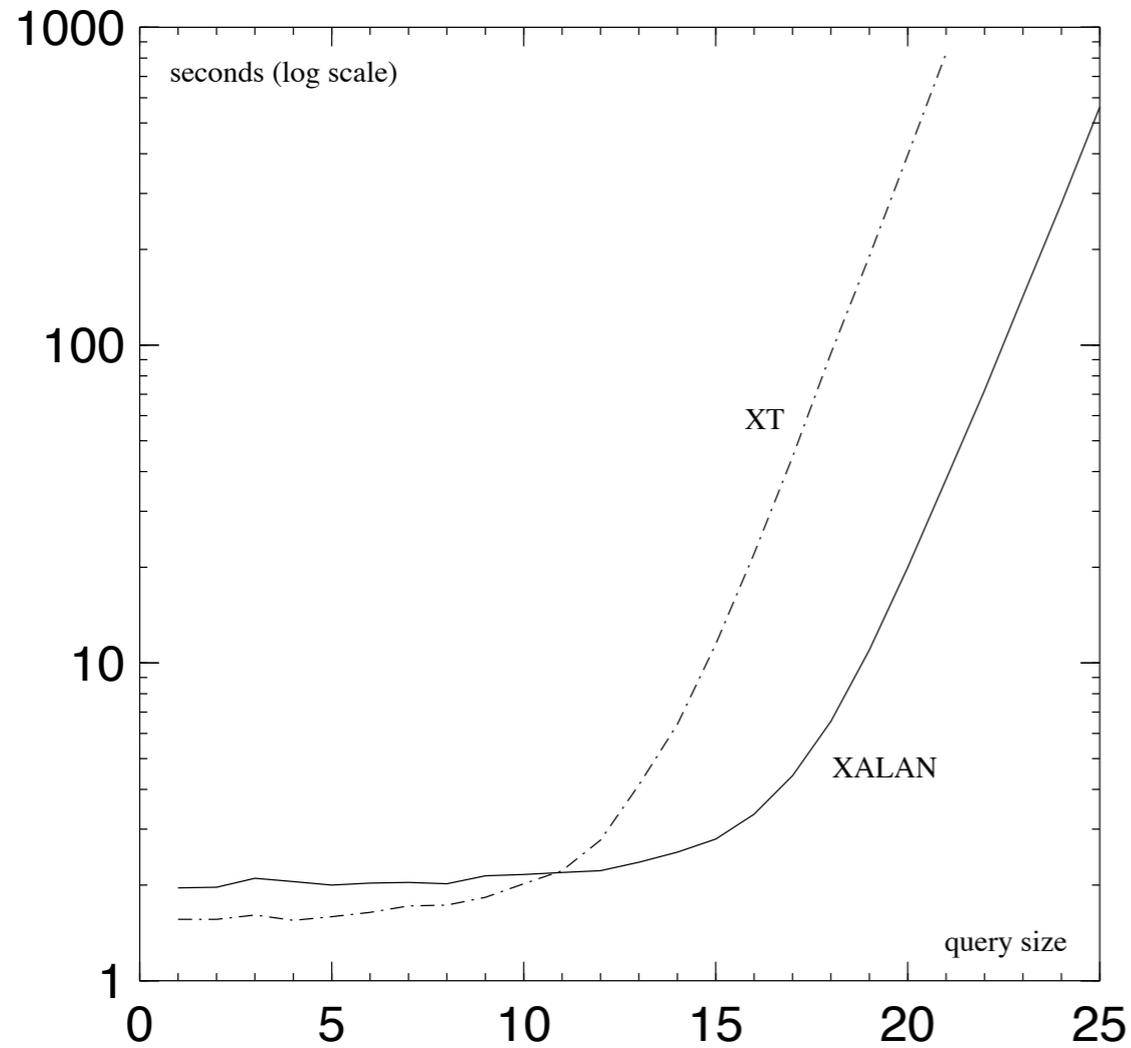
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université  
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# XPath: Performance

- XPath is a **navigational language** — specifies how the XML documents should be **traversed**
- **Main issue:** big volume of nodes can be extracted via XPath, so **efficient processing is still an ongoing challenge**

# XPath: Performance

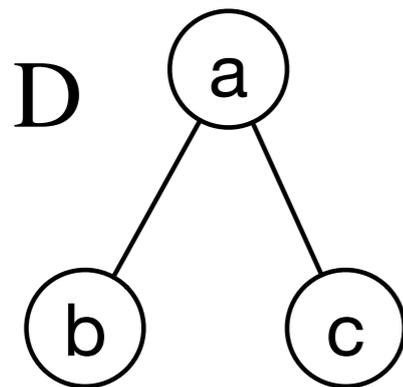


Gottlob, Koch, Pichler "Efficient Algorithms for Processing XPath Queries", VLDB 2002

# XPath: Performance

## Why?

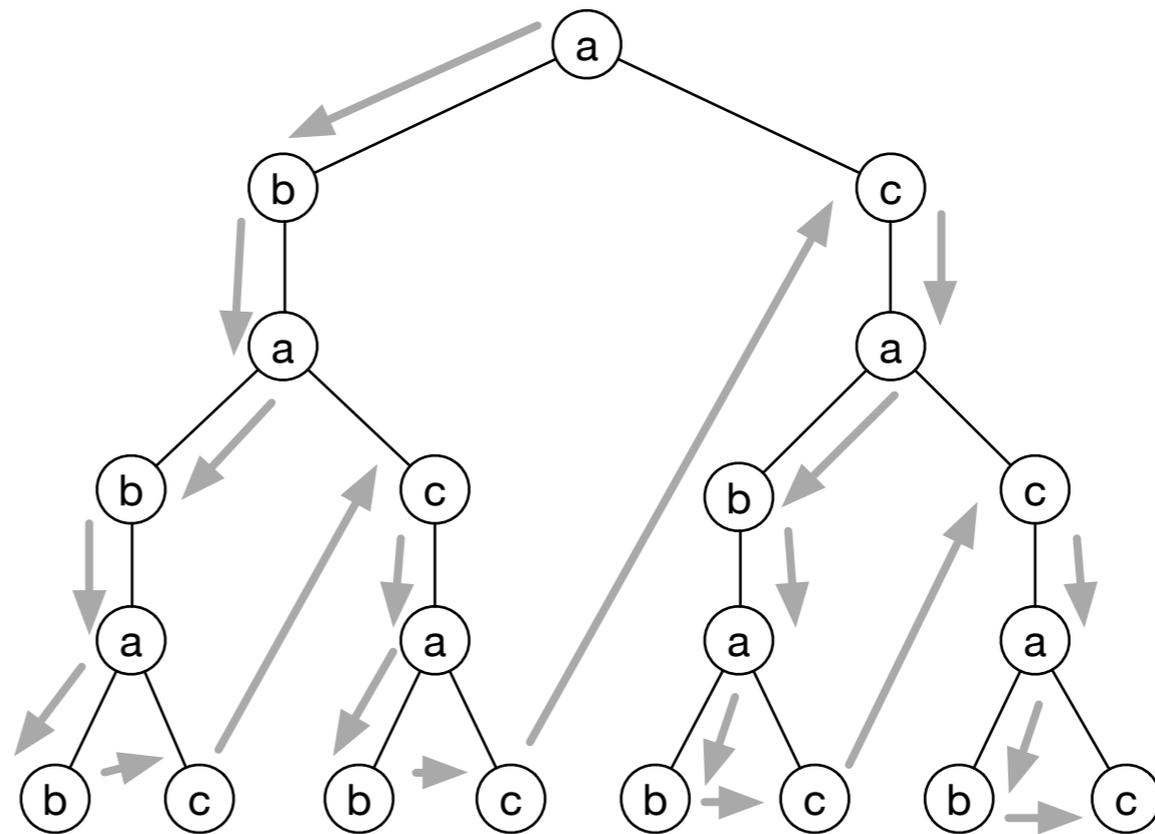
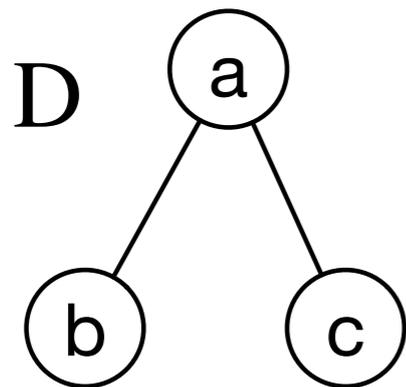
$Q := \text{child}::*/\text{parent}::*/\text{child}::*/\text{parent}::*/\text{child}::*$



# XPath: Performance

## Why?

$Q := \text{child}::*/\text{parent}::*/\text{child}::*/\text{parent}::*/\text{child}::*$



$$O(|D|^{|Q|})$$

# XPath: Performance

## Why?

```
procedure process-location-step( $n_0$ ,  $Q$ )  
  /*  $n_0$  is the context node;  
   query  $Q$  is a list of location steps */  
begin  
  node set  $S :=$  apply  $Q$ .first to node  $n_0$ ;  
  if ( $Q$ .tail is not empty) then  
    for each node  $n \in S$  do  
      process-location-step( $n$ ,  $Q$ .tail);  
     $O(|D|^{|Q|})$   
end
```

# Lecture Outline

- evaluating simple paths
- evaluating Core XPath
- evaluating Full XPath

# XPath: Simple Paths

- **Simple paths** are of the form:

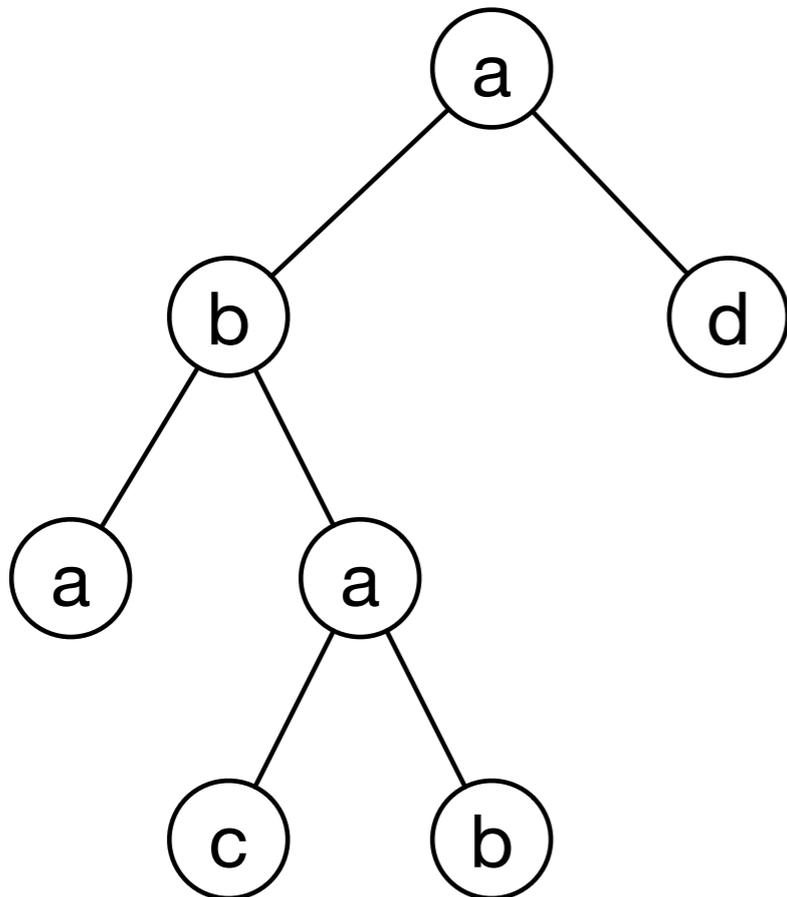
`//tag_1/tag_2/.../tag_n`

`//tag_1/tag_2/.../tag_n-1/text()`

- Can be evaluated in a **single pre-order traversal** (by using a stack)

# XPath: Simple Paths

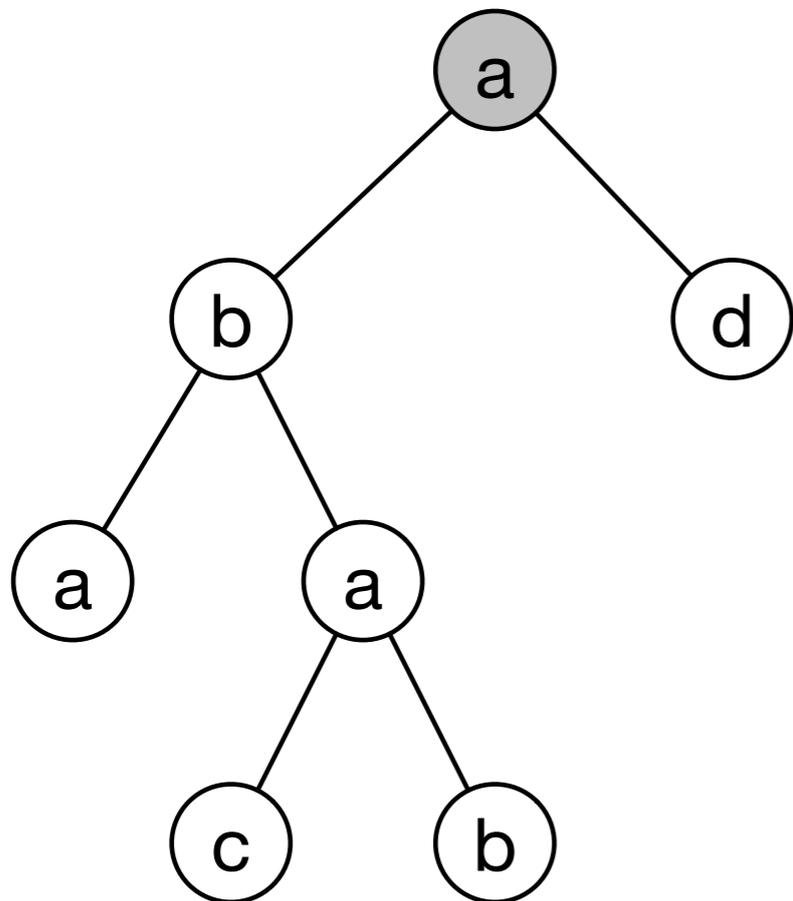
Q: *//a/b*



# XPath: Simple Paths

Q: **//a/b**

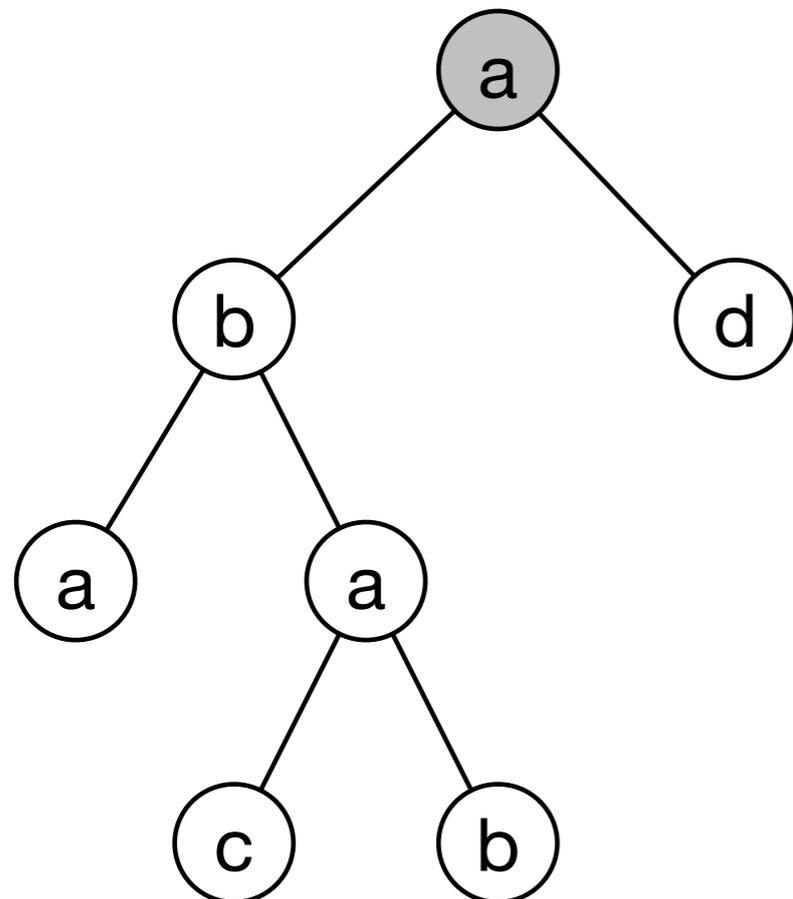
p = 1



# XPath: Simple Paths

Q: **//a/b**

$p = p+1 = 2$



Seq

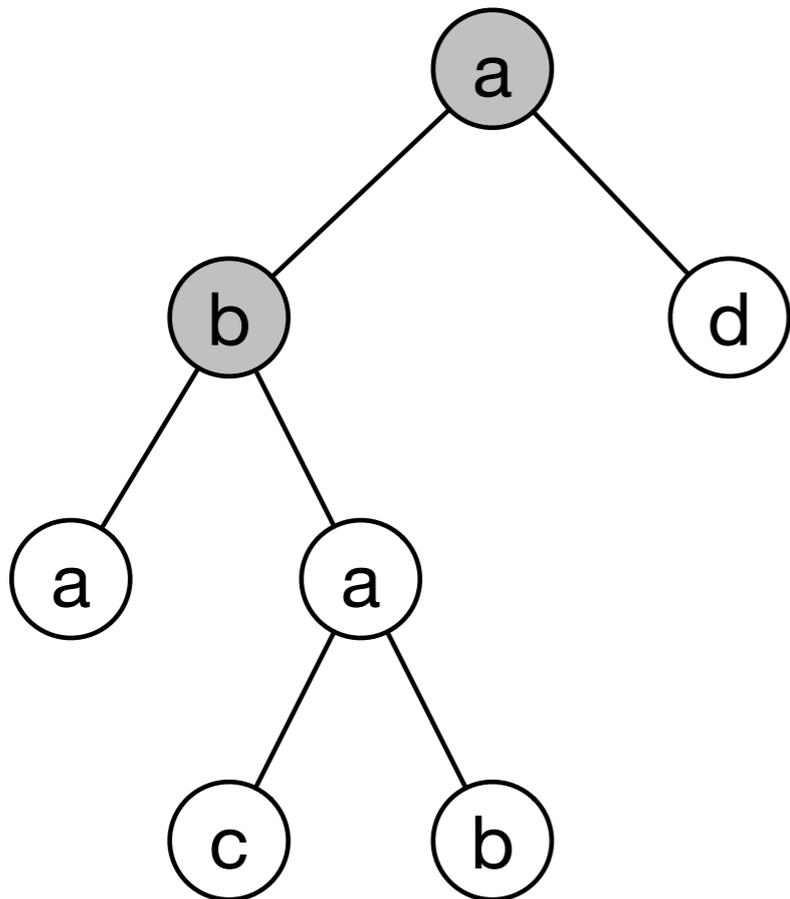
<a>

Stack

# XPath: Simple Paths

Q: **//a/b**

p = 2



Seq

<a>

<b>

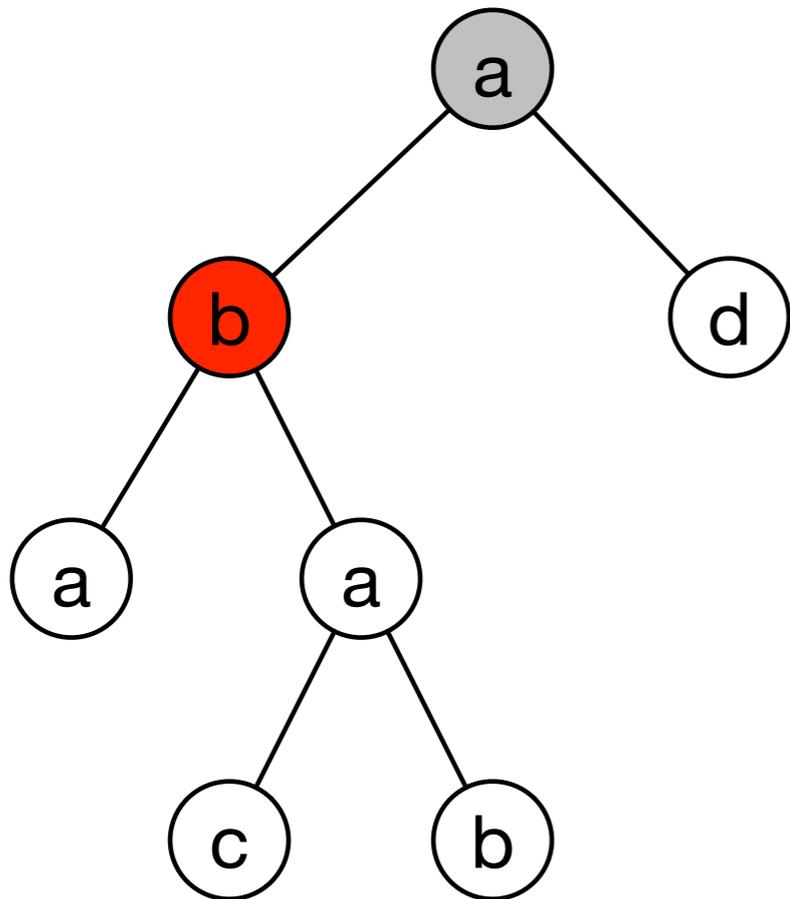
Stack

**Match!**

# XPath: Simple Paths

Q: `//a/b`

$p = 1$



Seq

<a>

<b>

Stack

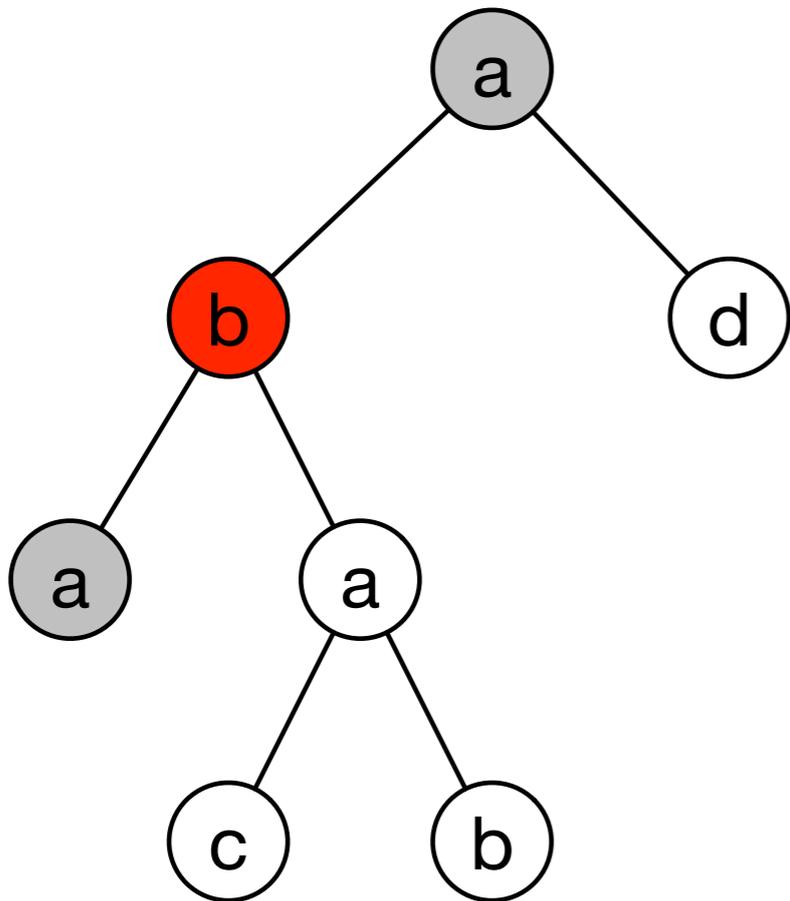
2

**Match!**

# XPath: Simple Paths

Q: **//a/b**

p = 1



Seq

<a>

<b>

<a>

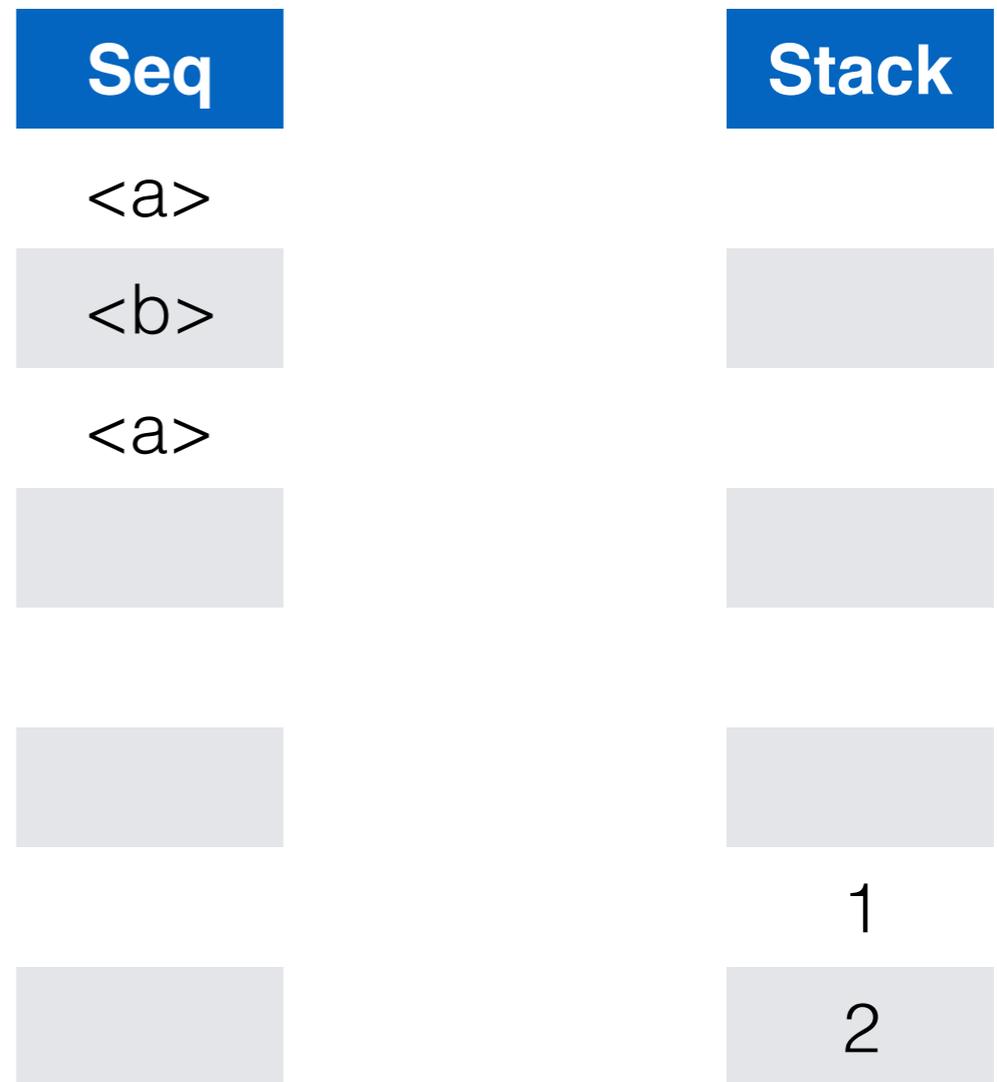
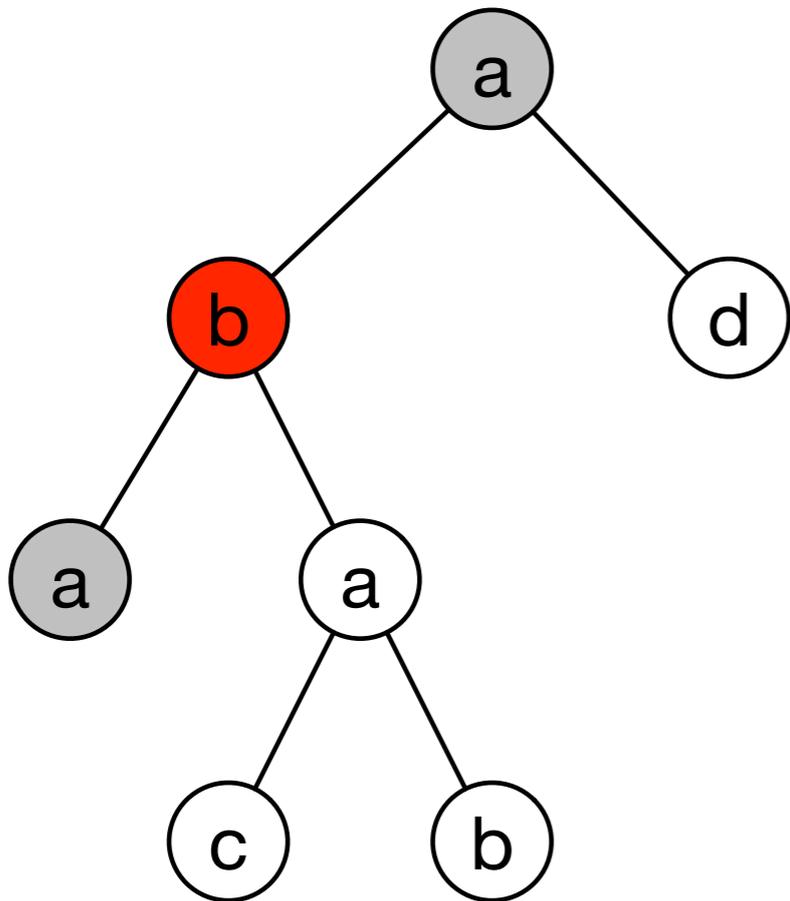
Stack

2

# XPath: Simple Paths

Q: **//a/b**

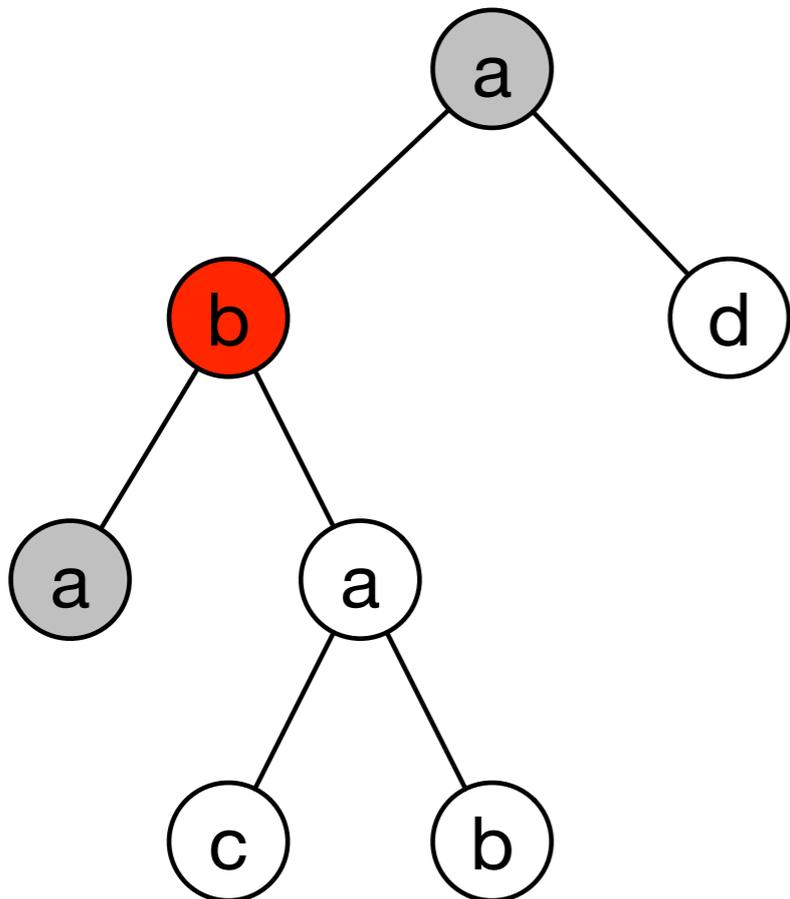
$p = p+1 = 2$



# XPath: Simple Paths

Q: **//a/b**

p = 2



Seq

<a>

<b>

<a>

</a>

Stack

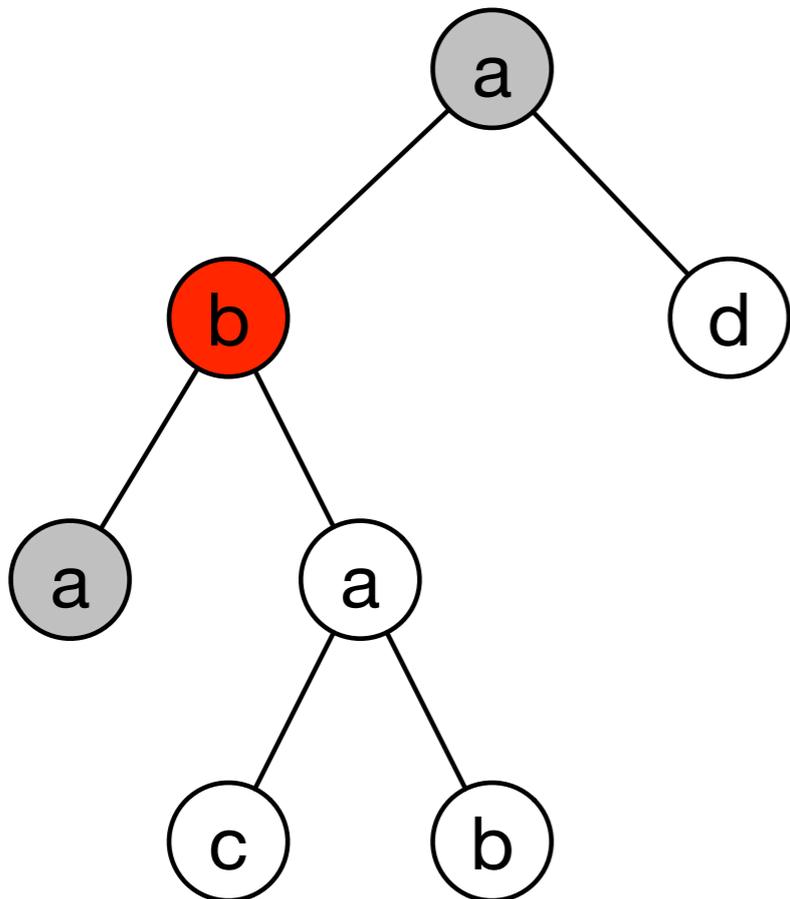
1

2

# XPath: Simple Paths

Q: **//a/b**

$p = \text{pop}() = 1$



**Seq**

<a>

<b>

<a>

</a>

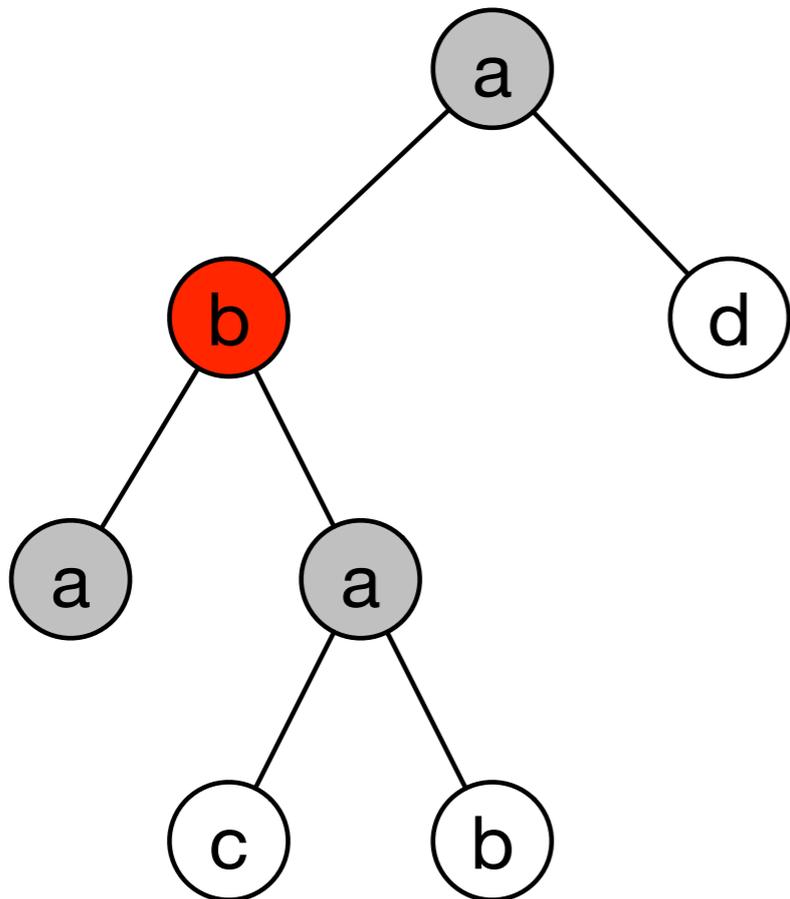
**Stack**

2

# XPath: Simple Paths

Q: **//a/b**

p = 1



Seq

<a>

<b>

<a>

</a>

<a>

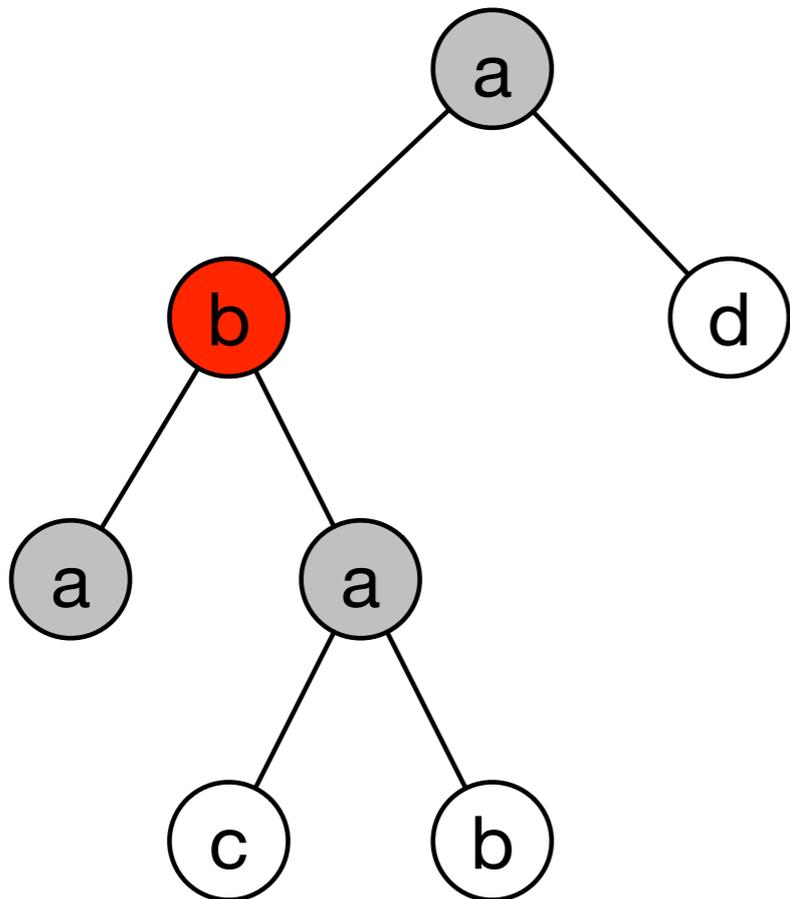
Stack

2

# XPath: Simple Paths

Q: **//a/b**

$p = p+1 = 2$

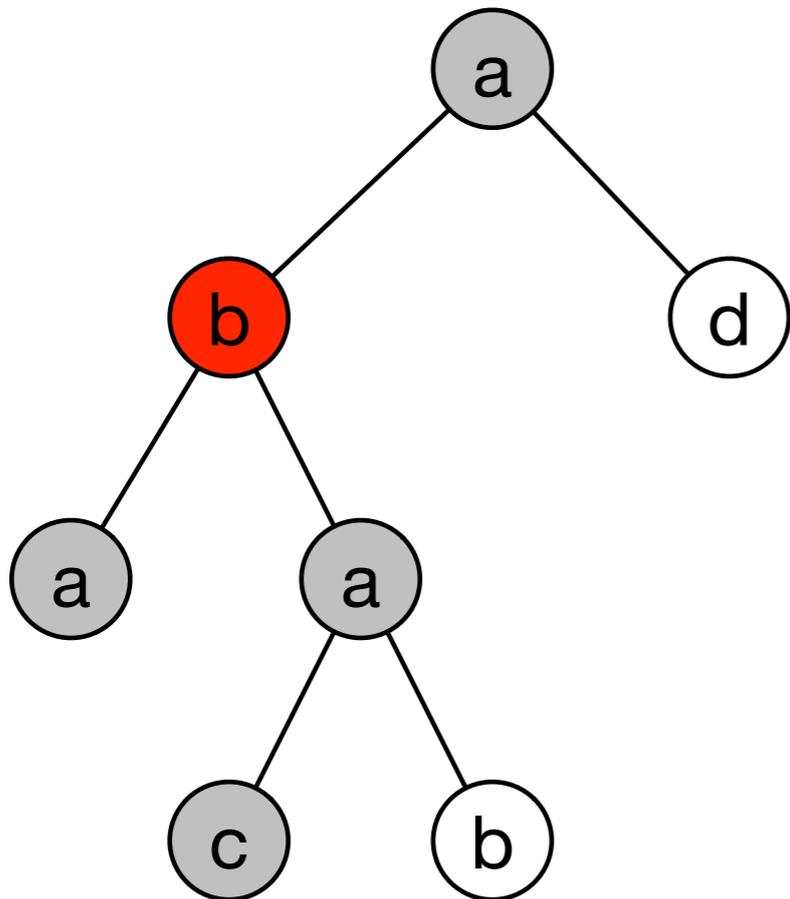


Seq	Stack
<a>	
<b>	
<a>	
</a>	
<a>	
	1
	2

# XPath: Simple Paths

Q: **//a/b**

p = 2



Seq

<a>

<b>

<a>

</a>

<a>

<c>

Stack

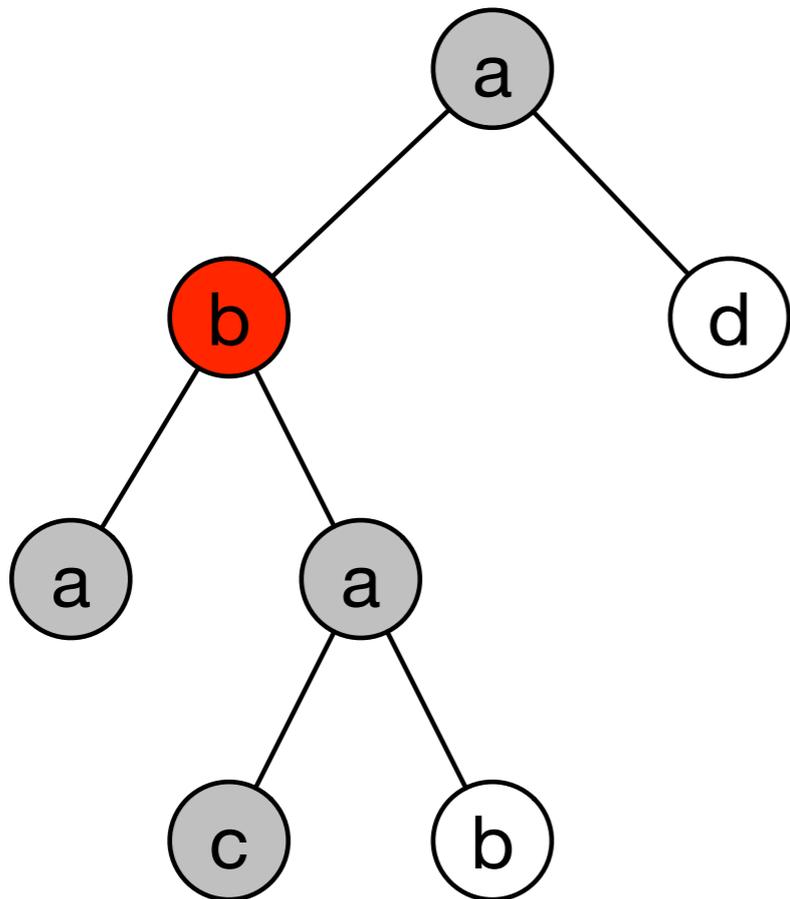
1

2

# XPath: Simple Paths

Q: **//a/b**

$p = \text{pop}() = 1$

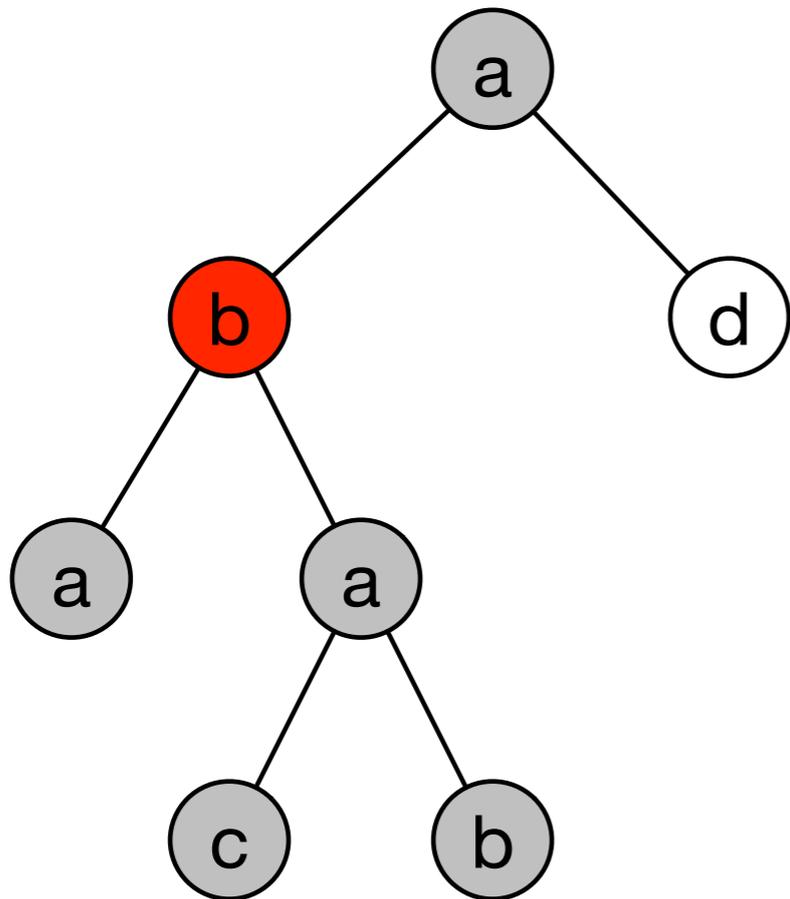


Seq	Stack
<a>	
<b>	
<a>	
</a>	
<a>	
<c>	
</c>	
	2

# XPath: Simple Paths

Q: **//a/b**

p = 2



Seq

<a>

<b>

<a>

</a>

<a>

<c>

</c>

<b>

Stack

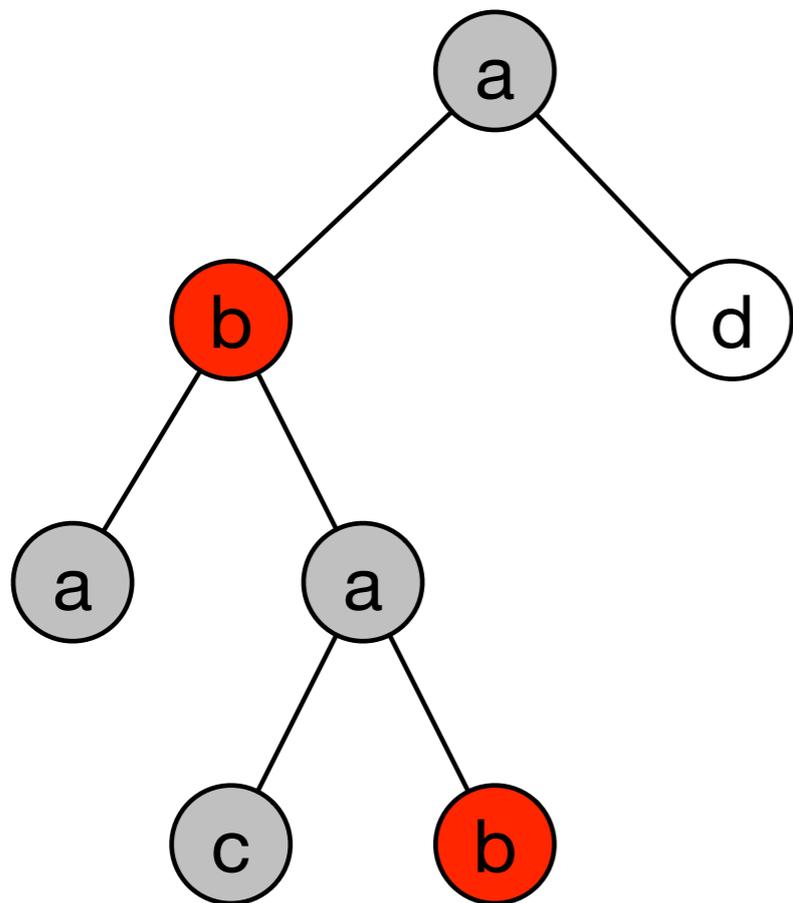
2

**Match!**

# XPath: Simple Paths

Q: **//a/b**

p = 1



Seq

<a>

<b>

<a>

</a>

<a>

<c>

</c>

<b>

Stack

2

**Match!**

# XPath: Simple Path Evaluation Complexity

- The algorithm is **linear in the size of the document**  $O(|D|)$
- Moreover, it can be implemented as a **streaming algorithm**
- Simple path evaluation can be **implemented on top of SAX** (Simple API for XML)

# XPath: Simple Path Evaluation In SAX

**Algorithm** (sketch):

1. **Initialization**: represent *path* query as an array for each step, maintain an array index *i* of the current step in the path, maintain a stack *S* of index positions
2. **startDocument**: empty stack *S*; *i*=1
3. **startElement**: if *path[i]* and element match, proceed to next step; otherwise, make a **failure transition**. Push *i* on *S*.
4. **endElement**: Pop old *i* from *S*.
5. **text**: If *path[i]=text*, we found a match. Otherwise, do nothing.

# Failure Transitions

*Example:*

Q: //a/b/a/c/ but we have seen //a/b/a/**b**

- postfix of we have seen is prefix of the query!

Q: //a/b/a/c/                      //a/b/a/b

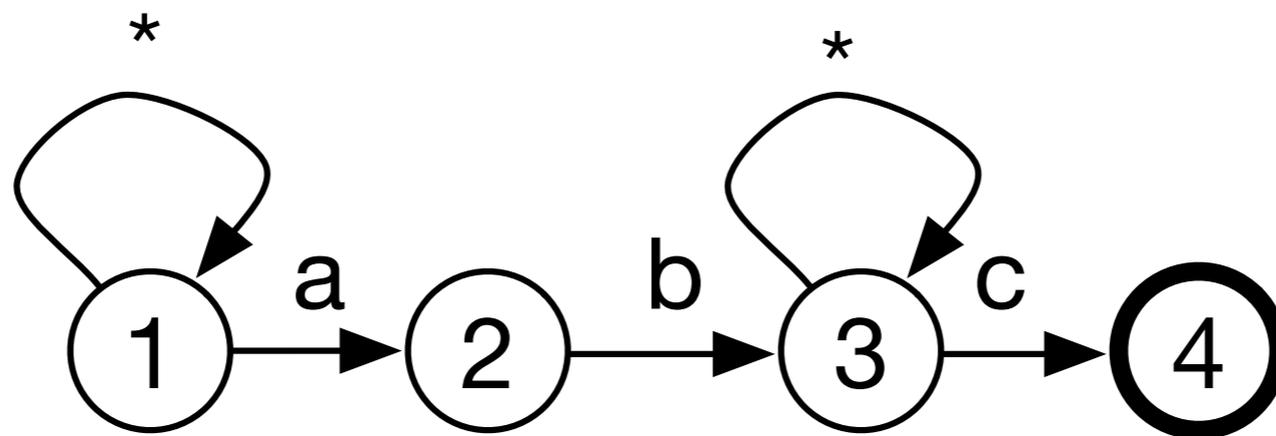
- this can be done via the [Knuth-Morris-Pratt algorithm](#) — linear string matching

# Evaluation Using Automata

**Principle:** Use the XPath expression as a regular expression matching the paths of the tree.

# Evaluation Using Automata

//a/b//c



# Evaluation Using Automata

- dealing with \* transitions is quite tricky
- transforming the NFA into a DFA has exponential blow-up
- good news: do not need to transform into DFA (lazy DFA)

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- dealing with \* transitions is quite tricky
- transforming the NFA into a DFA has **exponential blow-up**
- **good news**: do not need to transform into DFA (lazy DFA)

Green, Gupta, Miklau, Onizuka, Suciu. “**Processing XML Streams with Deterministic Automata and Stream Indexes**”  
ACM TODS 29(4), 2004

# XPath: Core XPath

Core XPath contains:

- all **12 axes**
- all **node tests** (only element nodes)
- filters with **logical operators: and, or, not**

# XPath: Bottom-up Evaluation of Core XPath

Set operations on nodes:

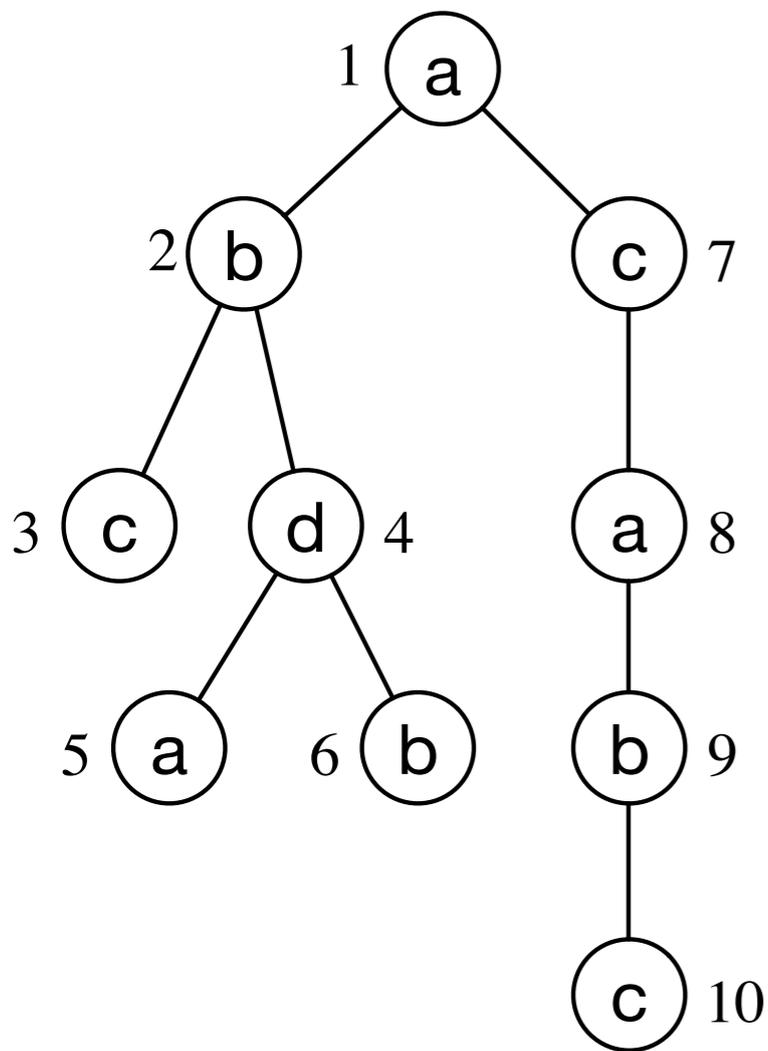
Operation	Objective
$\text{axis}(S_1) = S_2$	the node ids corresponding to the axis axis
$\cap(S_1, S_2) = S_3$	intersection of sets; for steps and <b>and</b>
$\cup(S_1, S_2) = S_3$	union of sets; for <b>or</b>
$- (S_1, S_2) = S_3$	difference of sets; for <b>not</b>
$T(\text{label}) = S_1$	set of node ids labelled label

# XPath: Bottom-up Evaluation of Core XPath

**Algorithm** (sketch):

1. Transform the query into a tree composed of set operations;
2. Starting at the root (or at the filters); evaluate the set operations bottom-up;
3. The final results are the nodes corresponding to node ids.

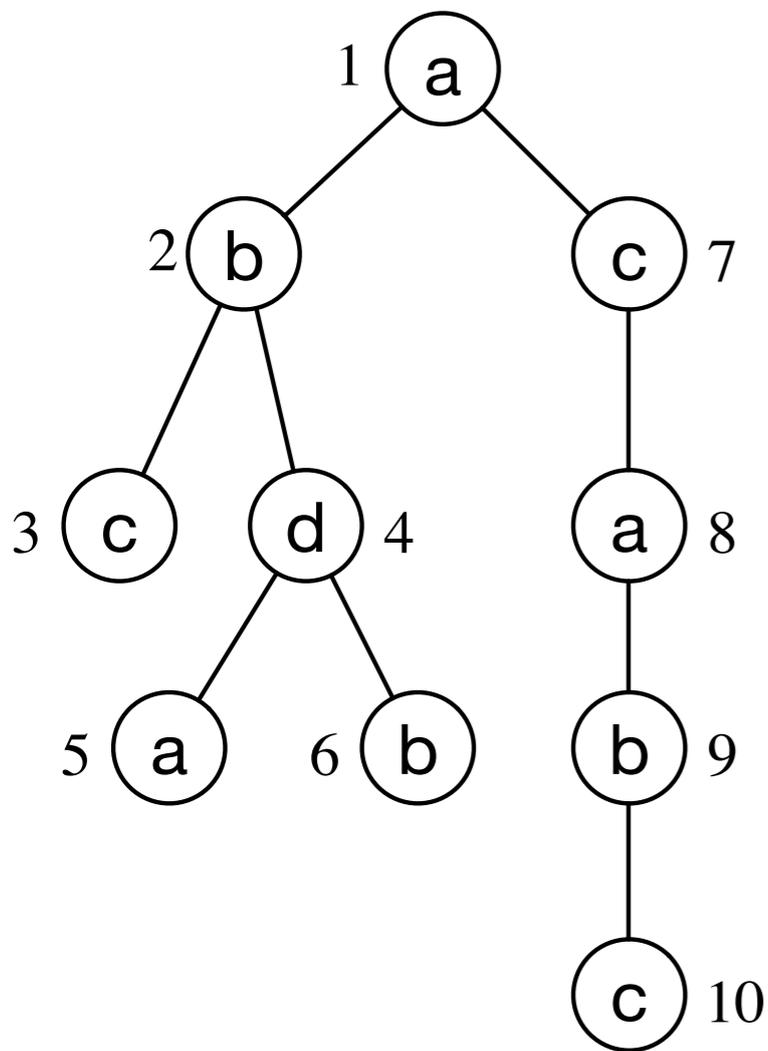
# XPath: Bottom-up Evaluation of Core XPath



label	T(label)
a	{1,5,8}
b	{2,6,9}
c	{3,7,10}
d	{4}

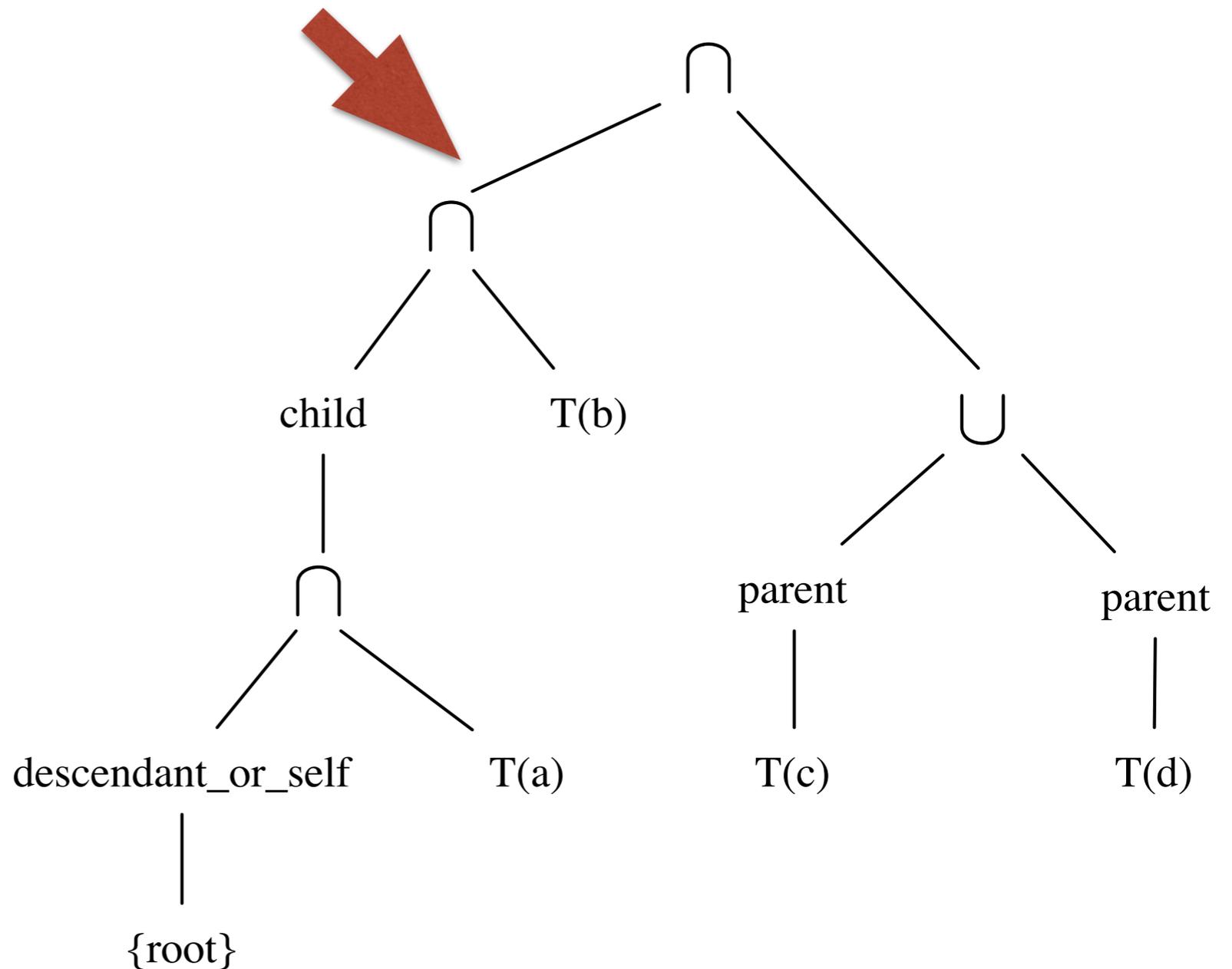
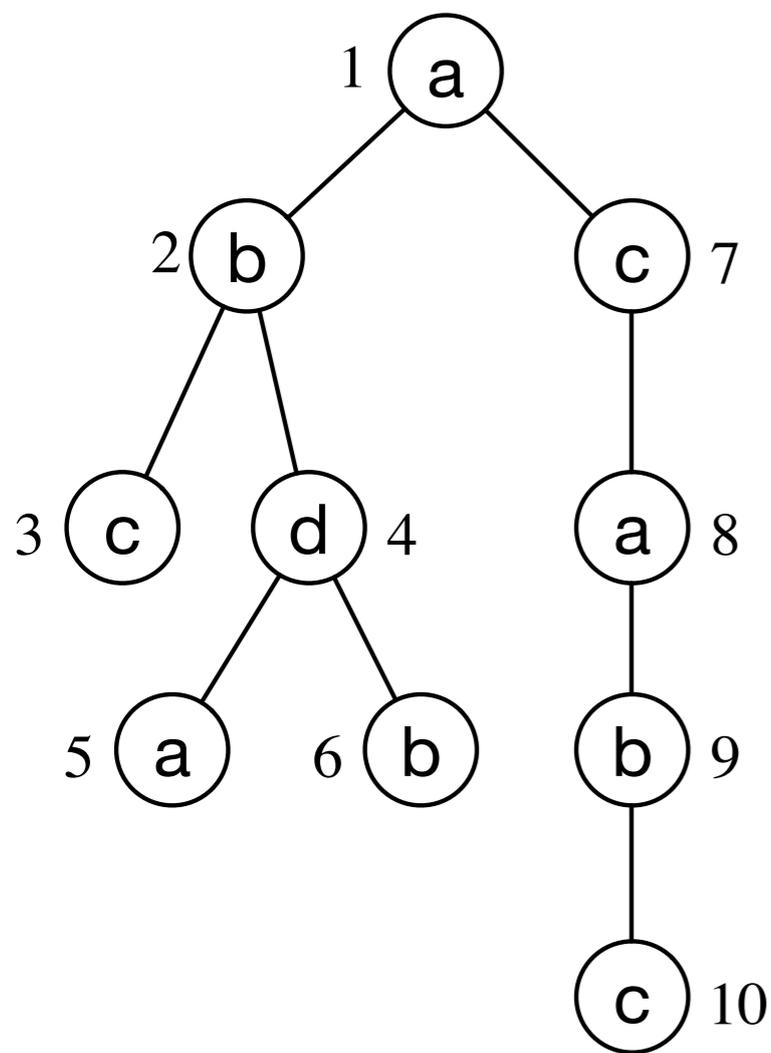
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`/descendant_or_self::a/child::b[child::d or child::c]`



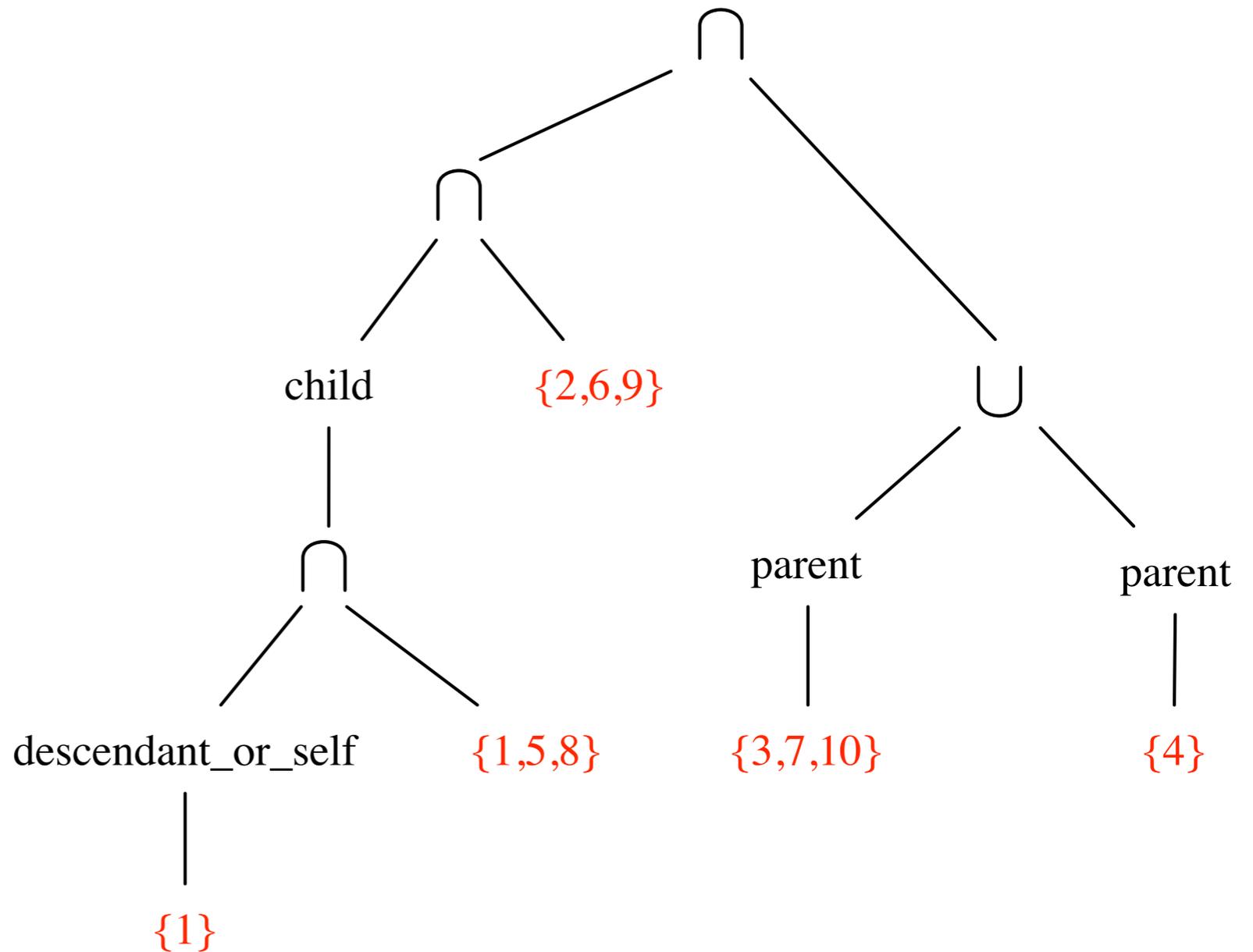
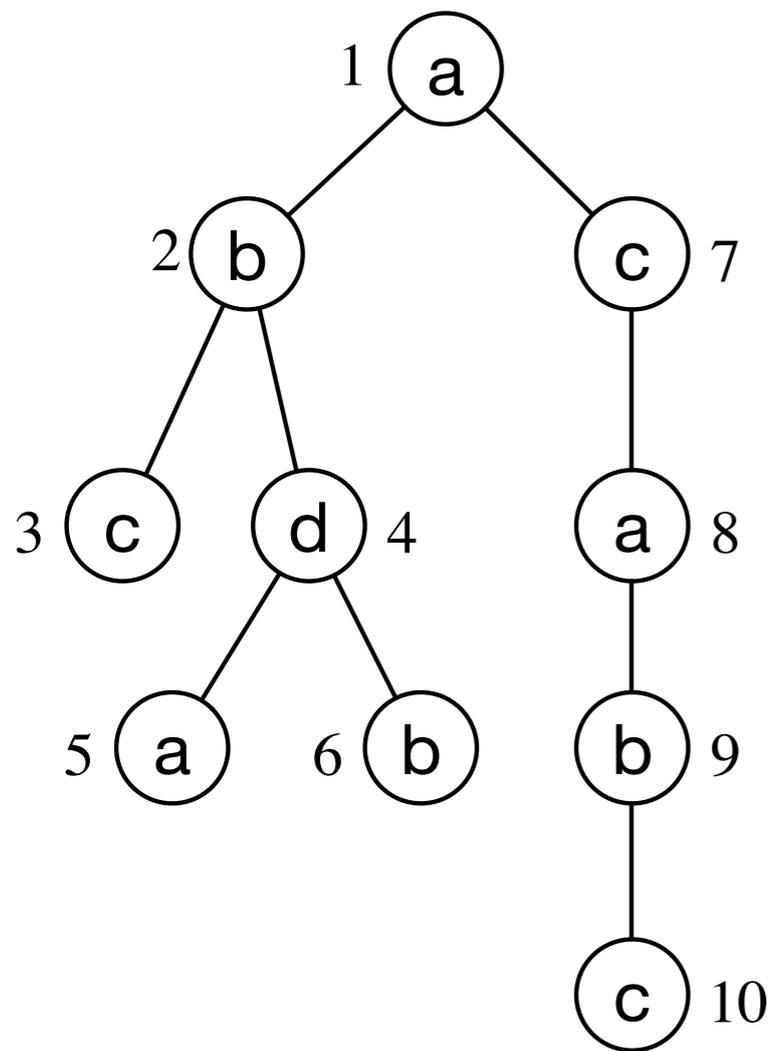
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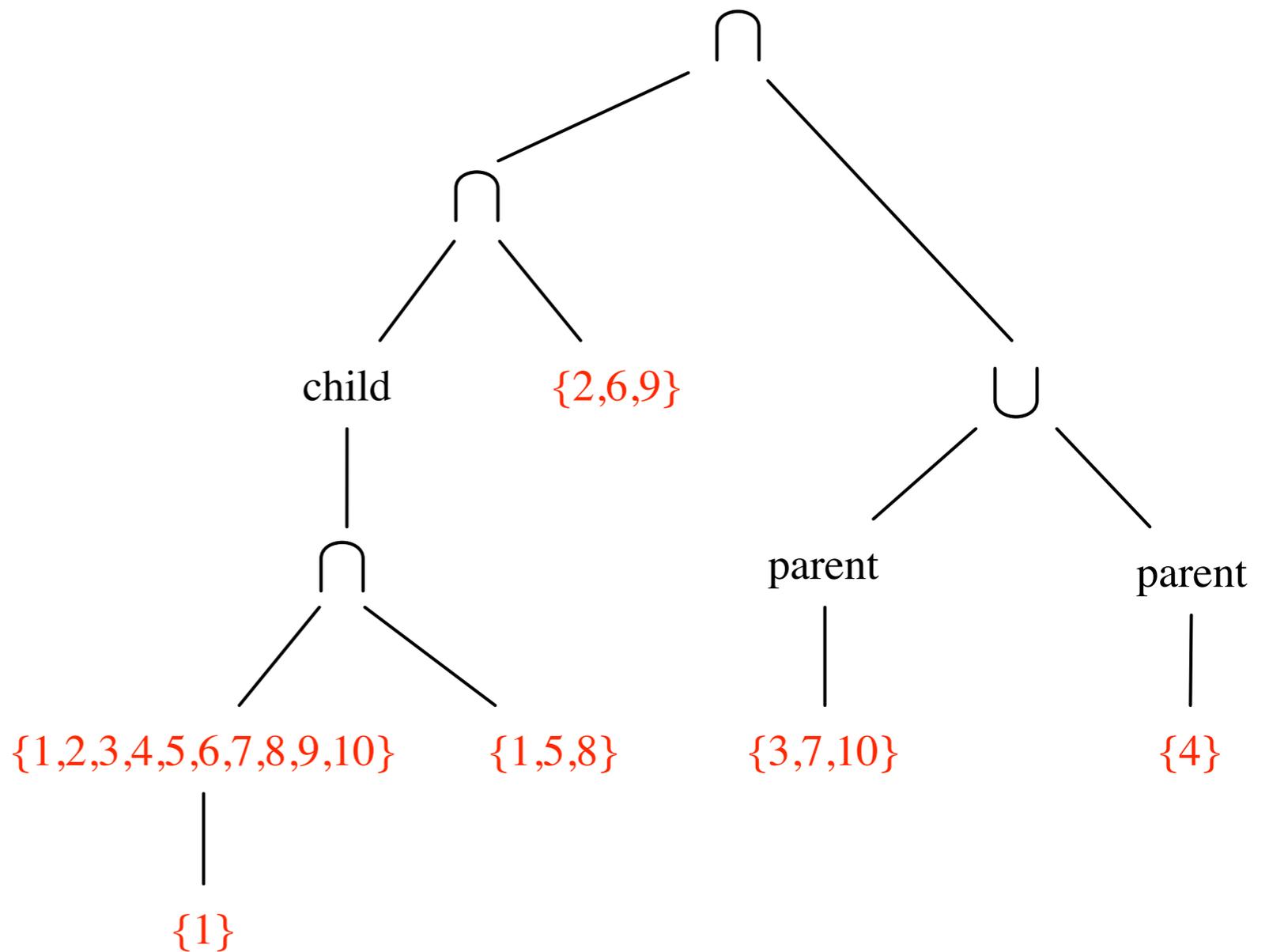
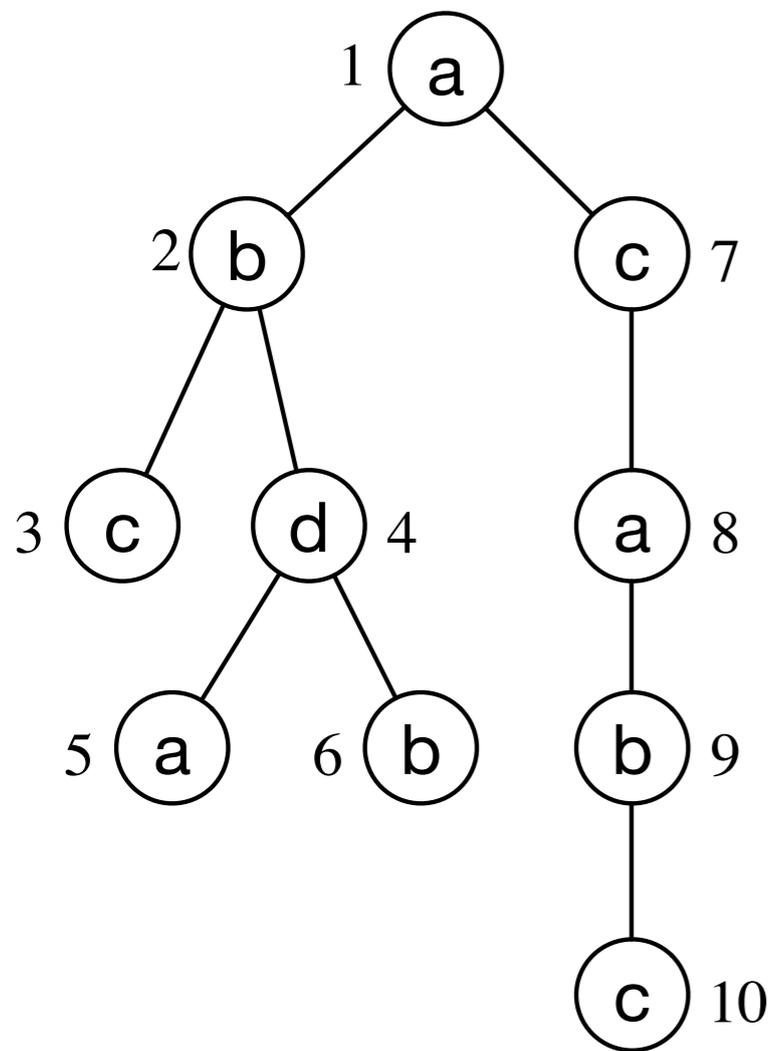
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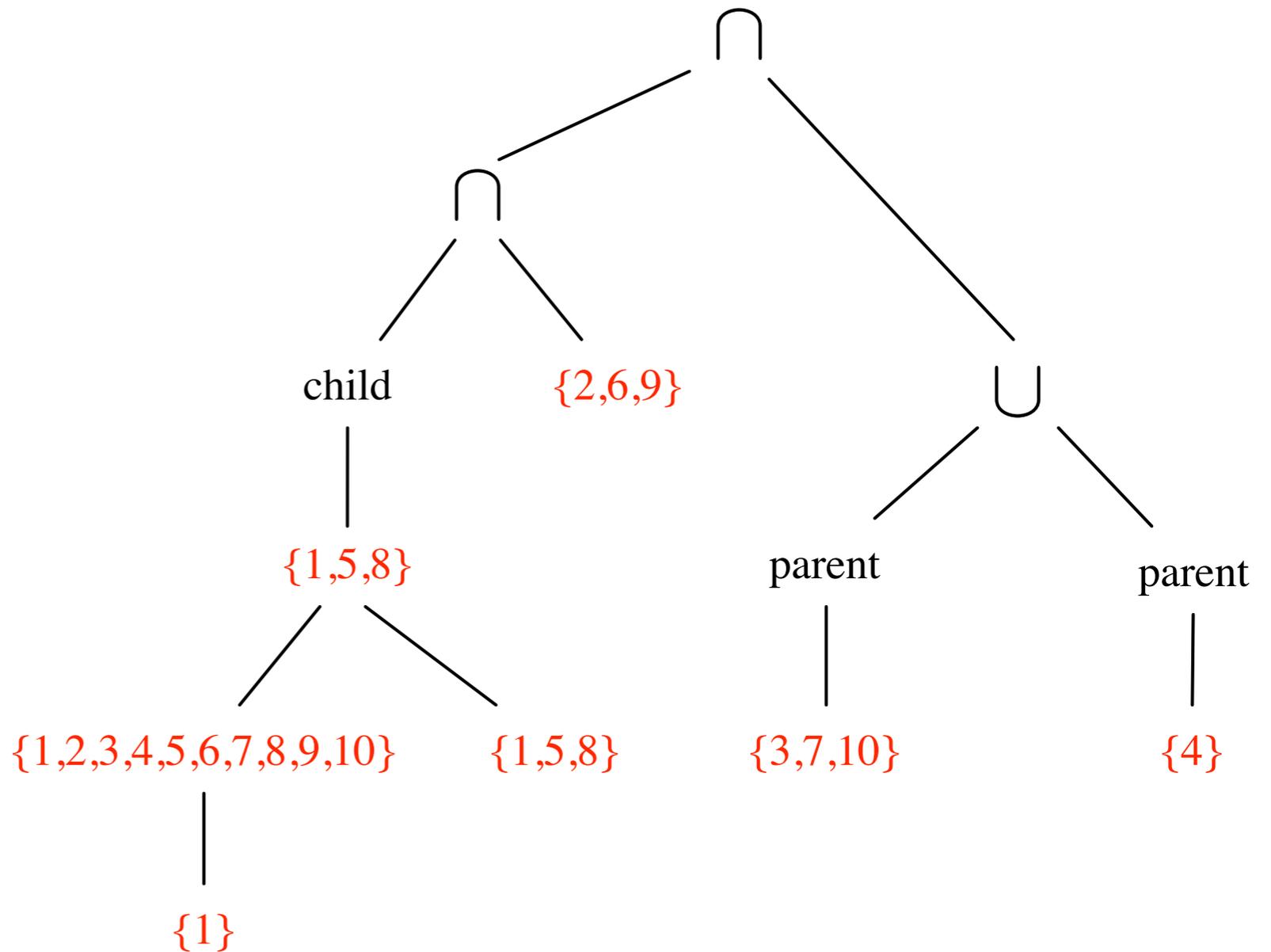
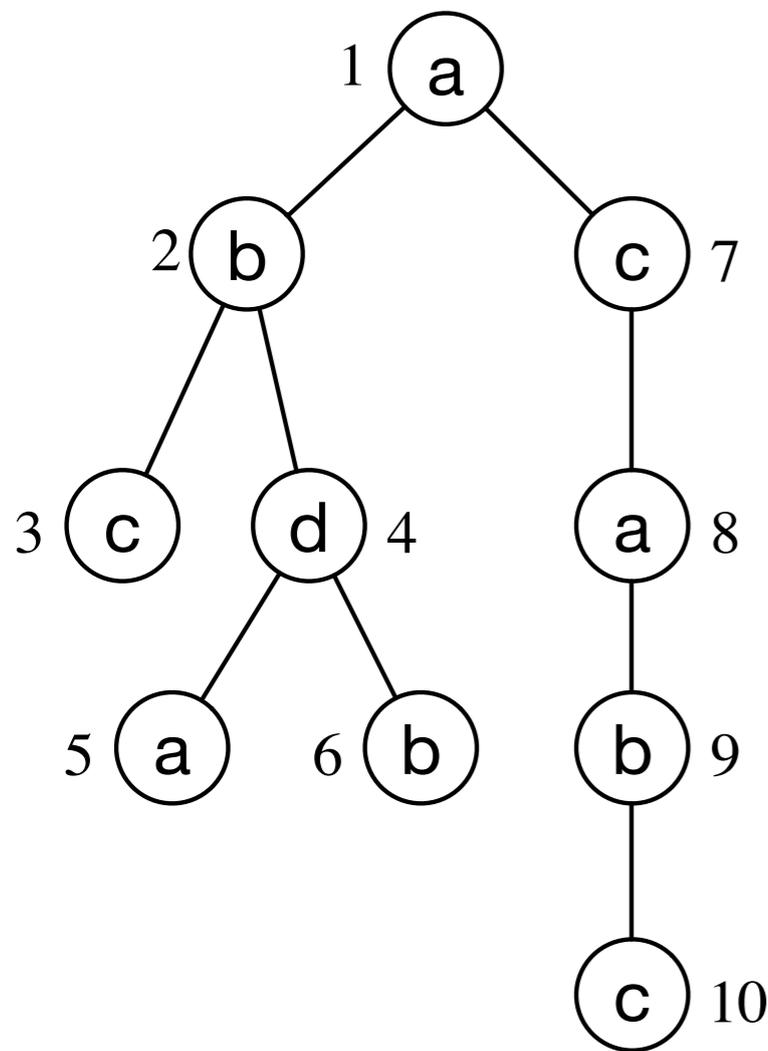
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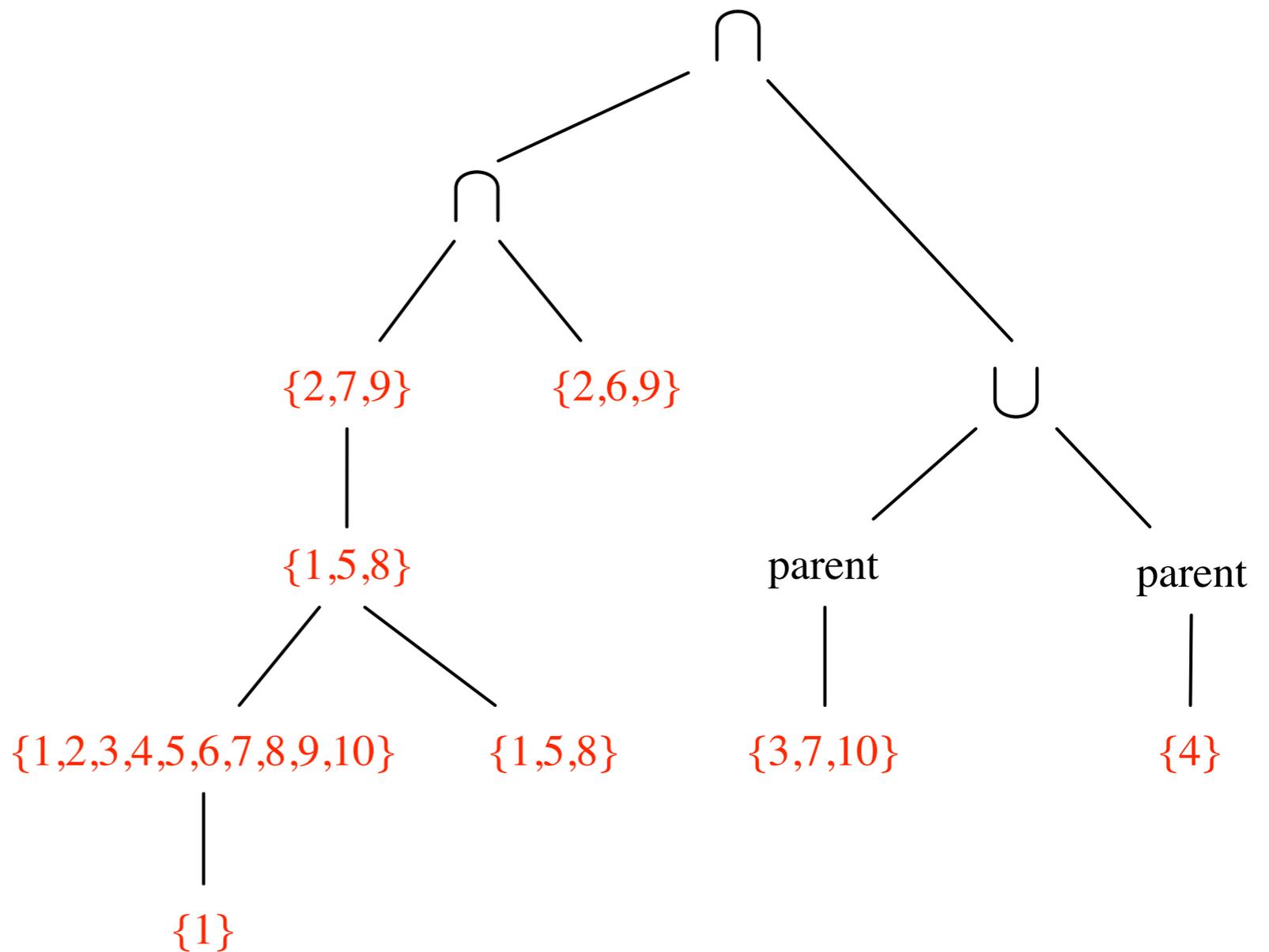
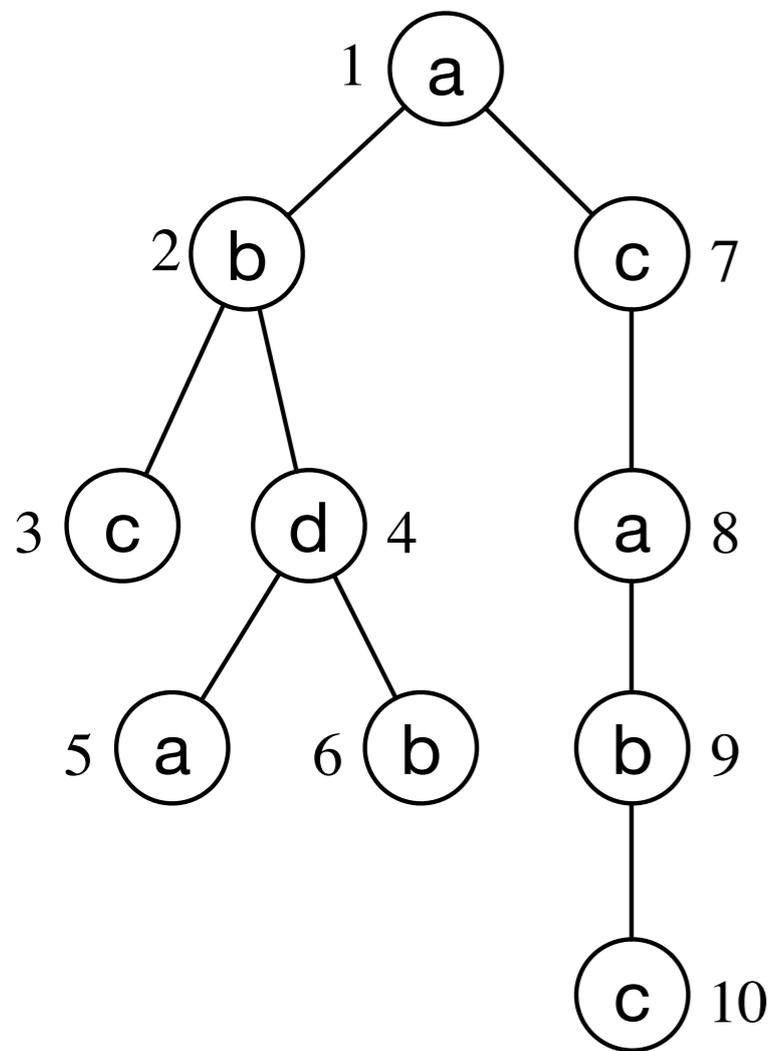
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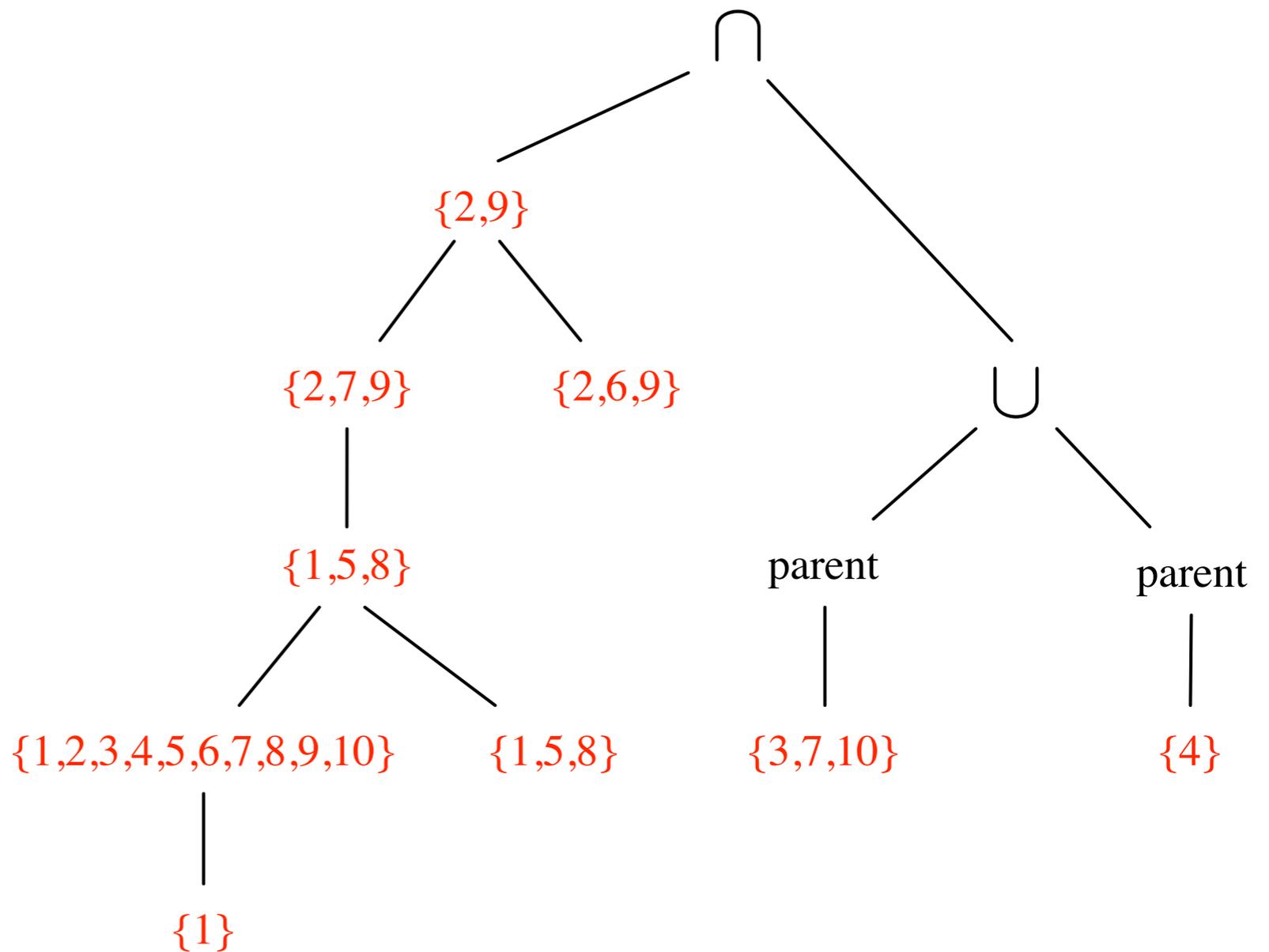
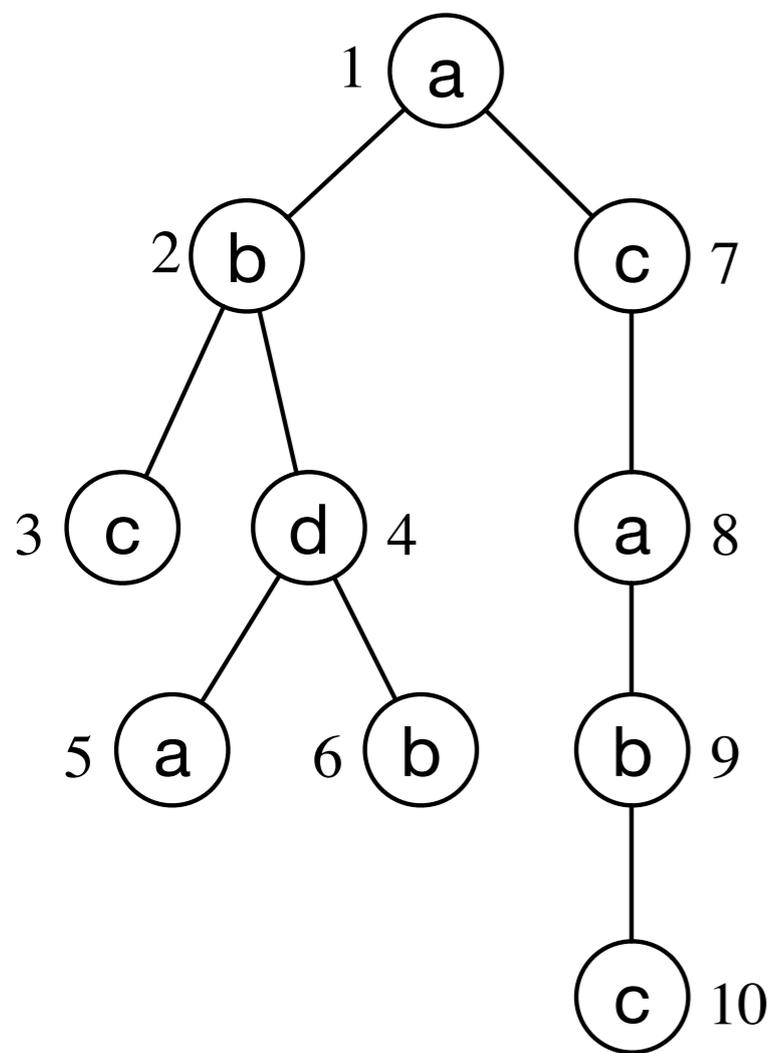
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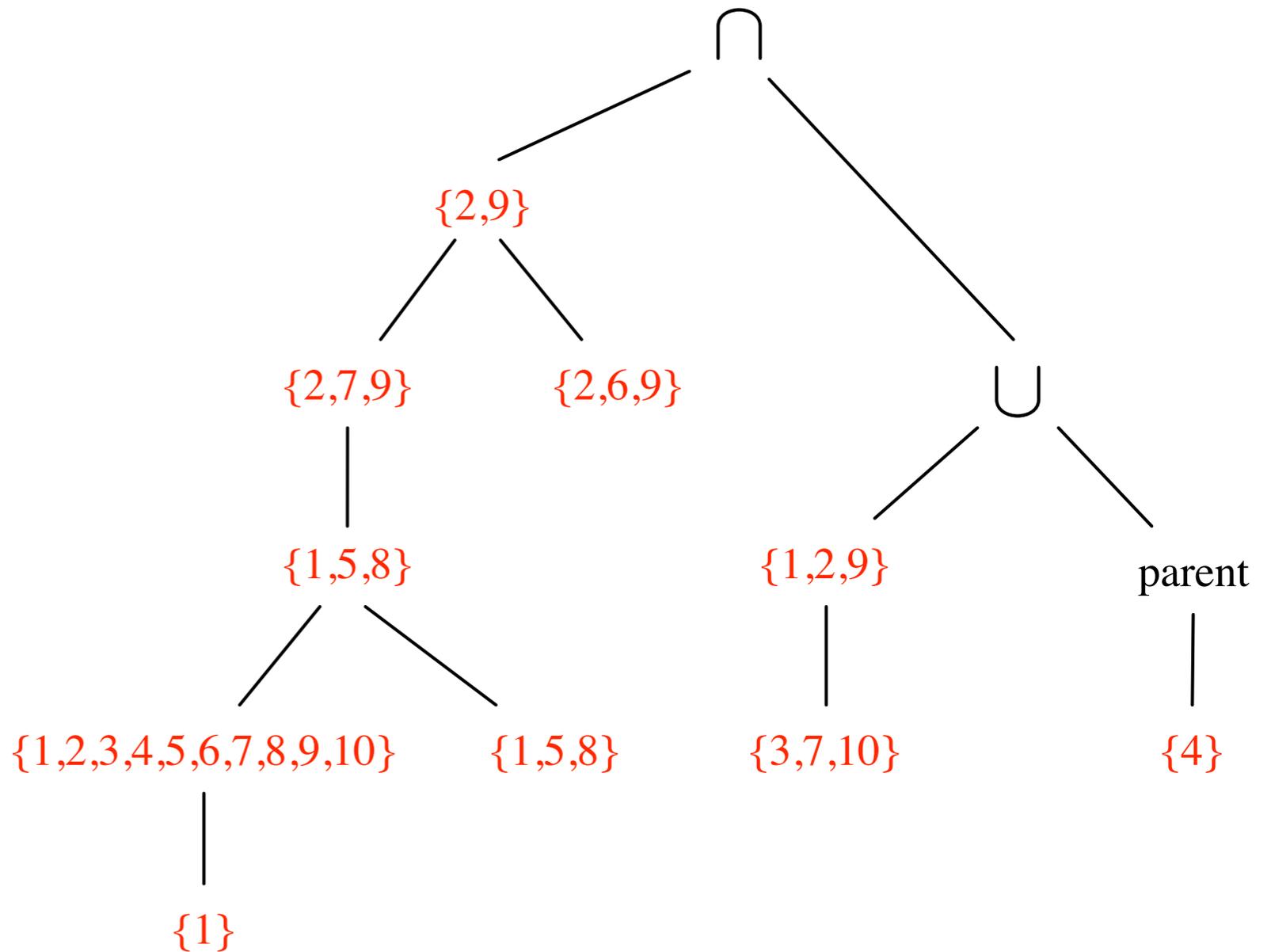
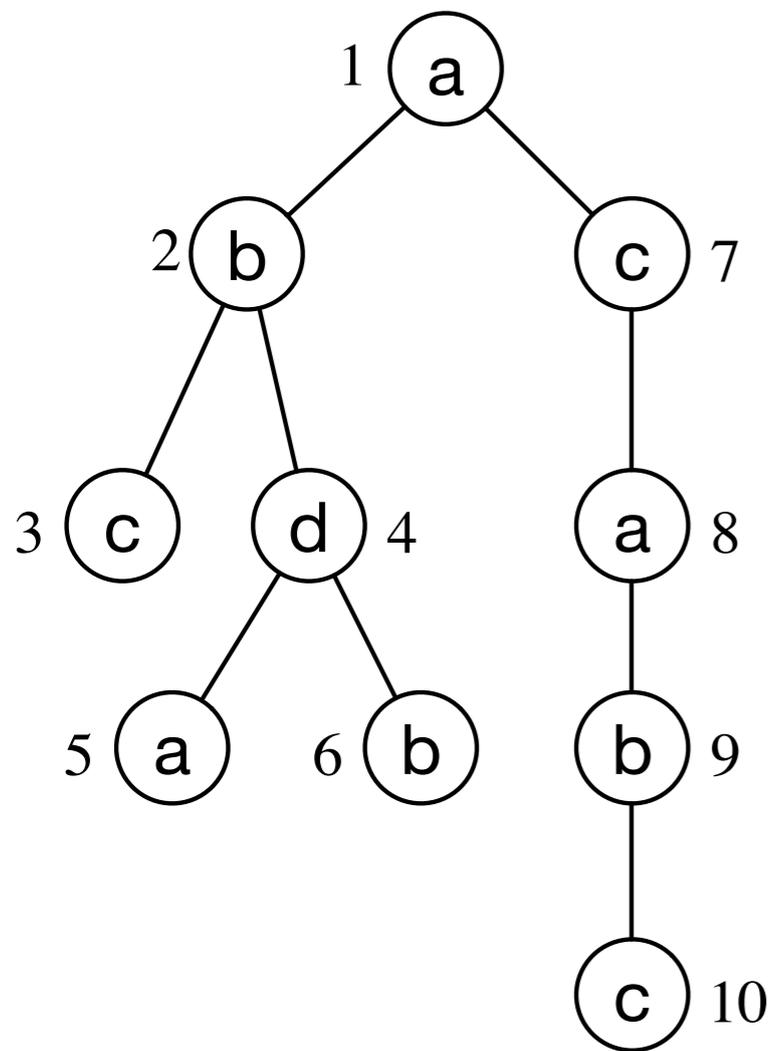
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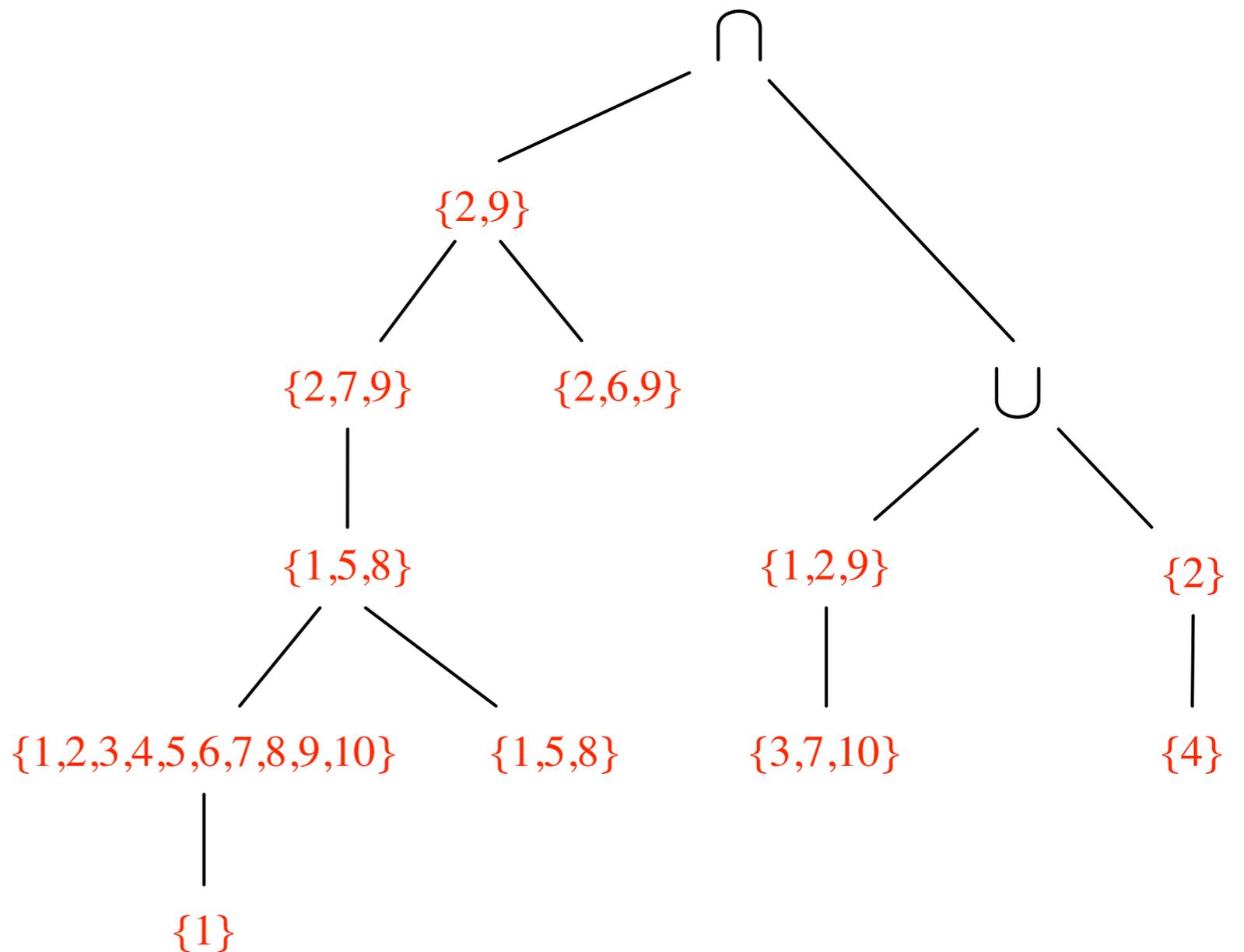
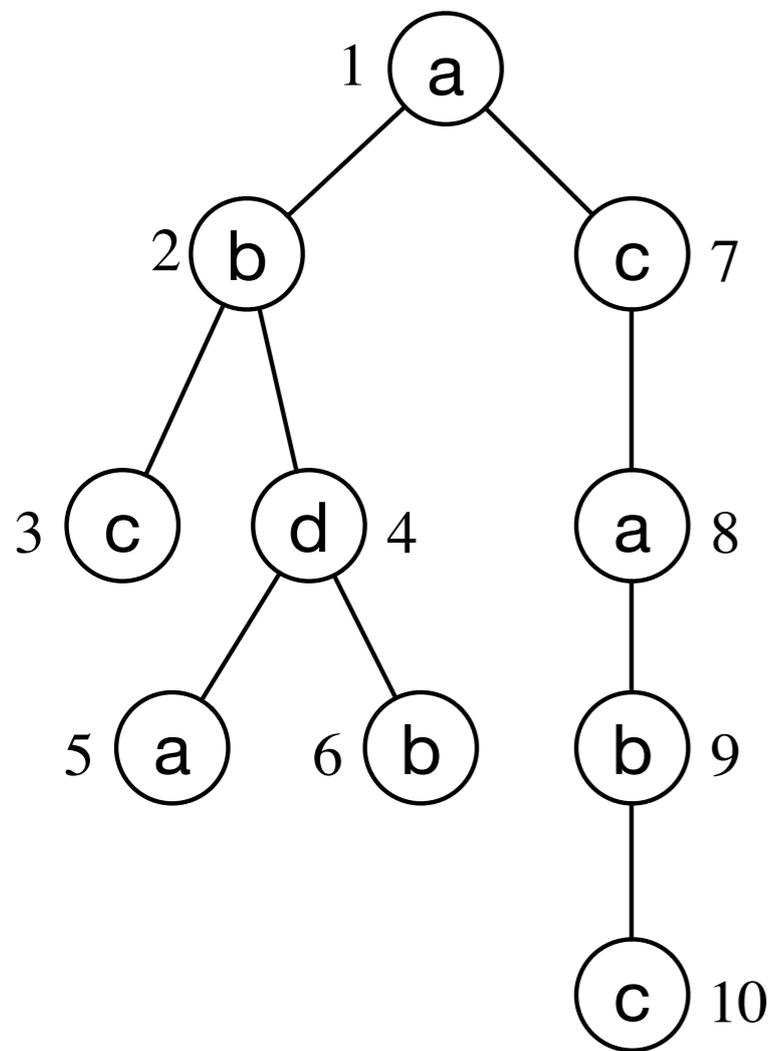
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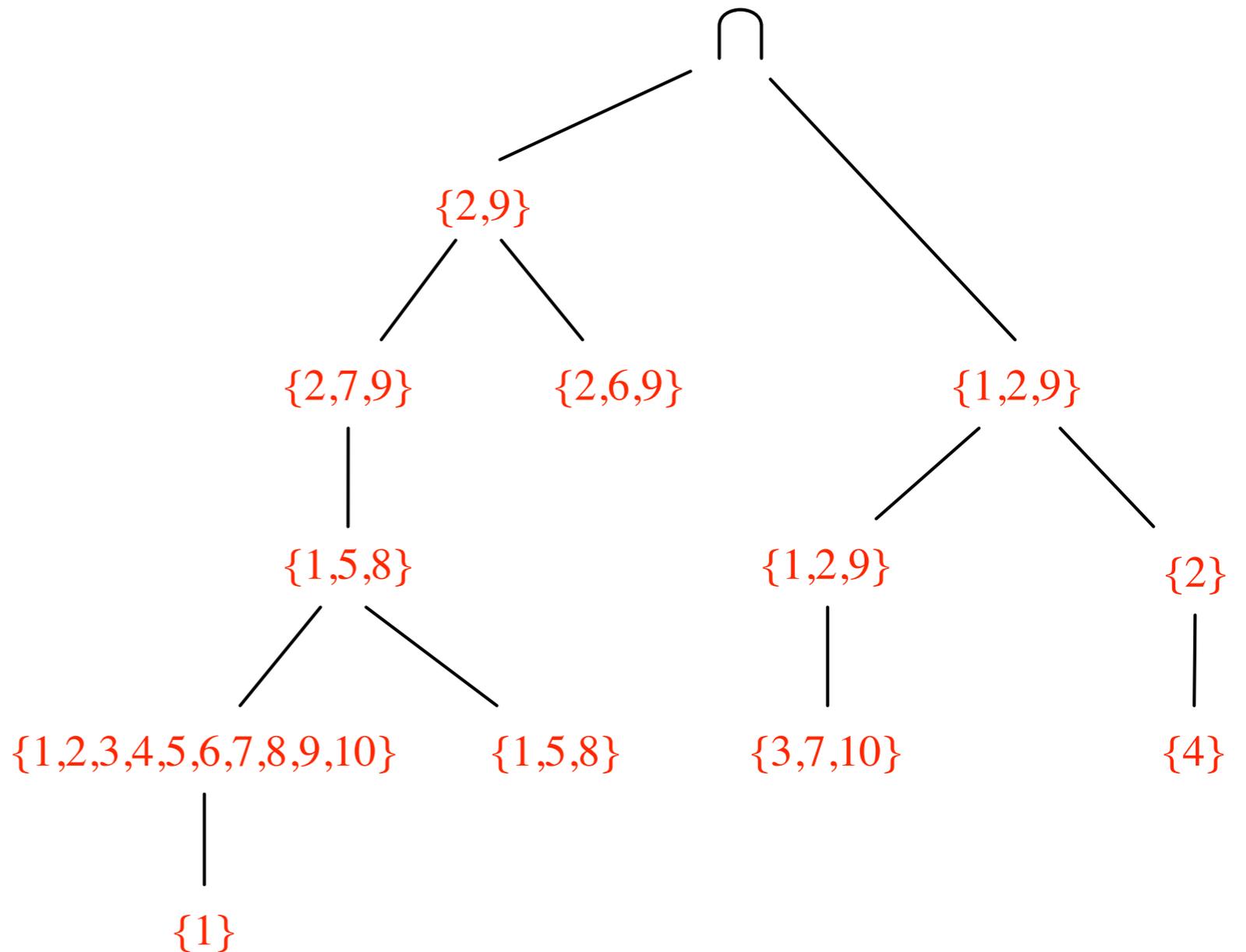
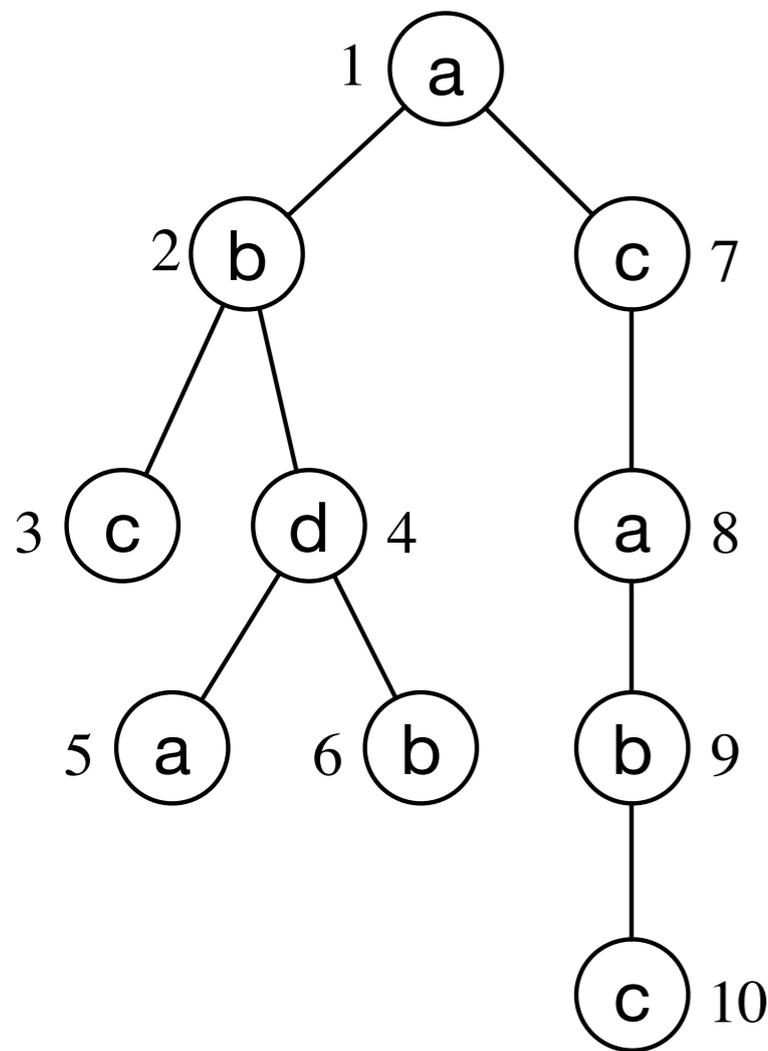
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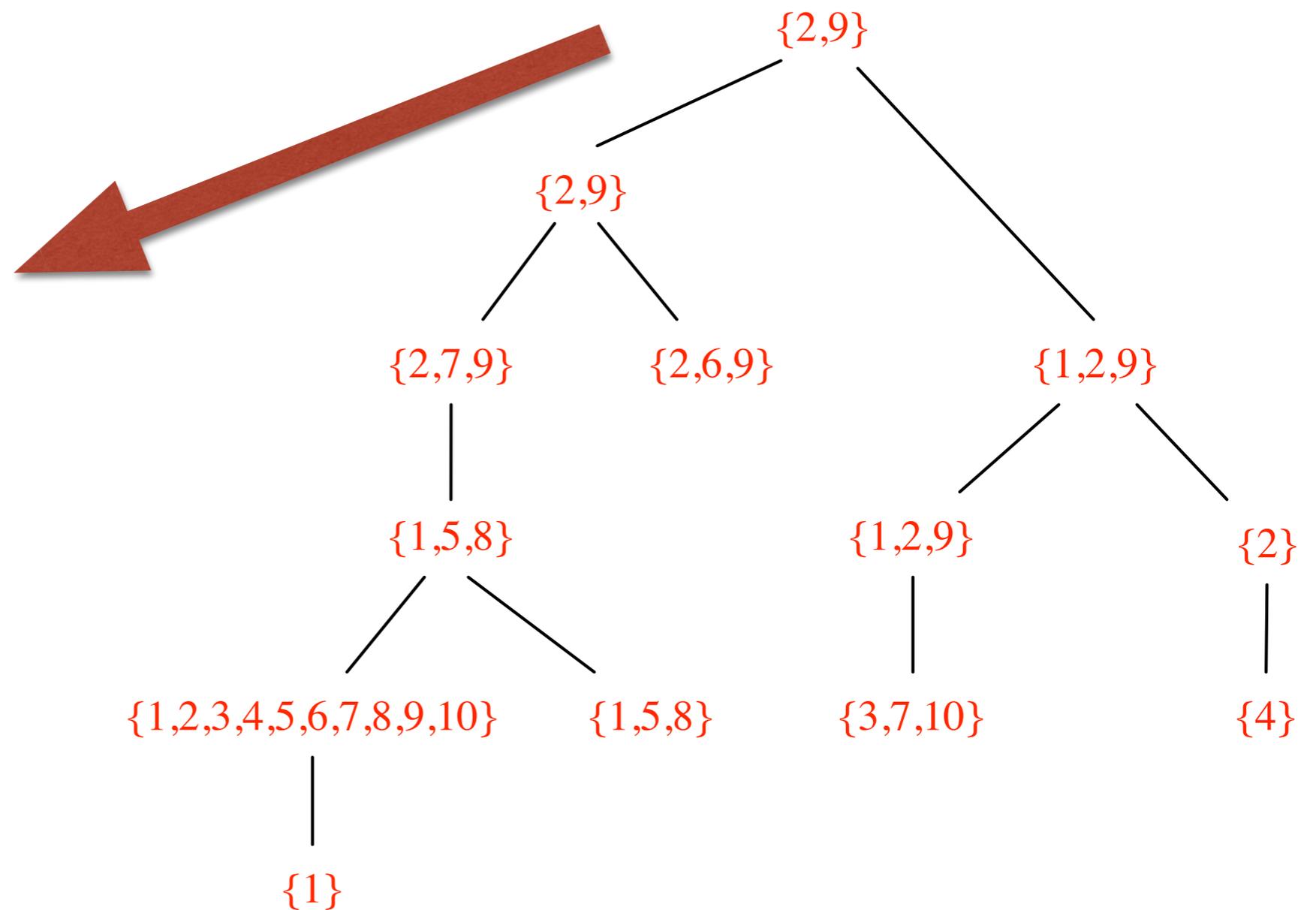
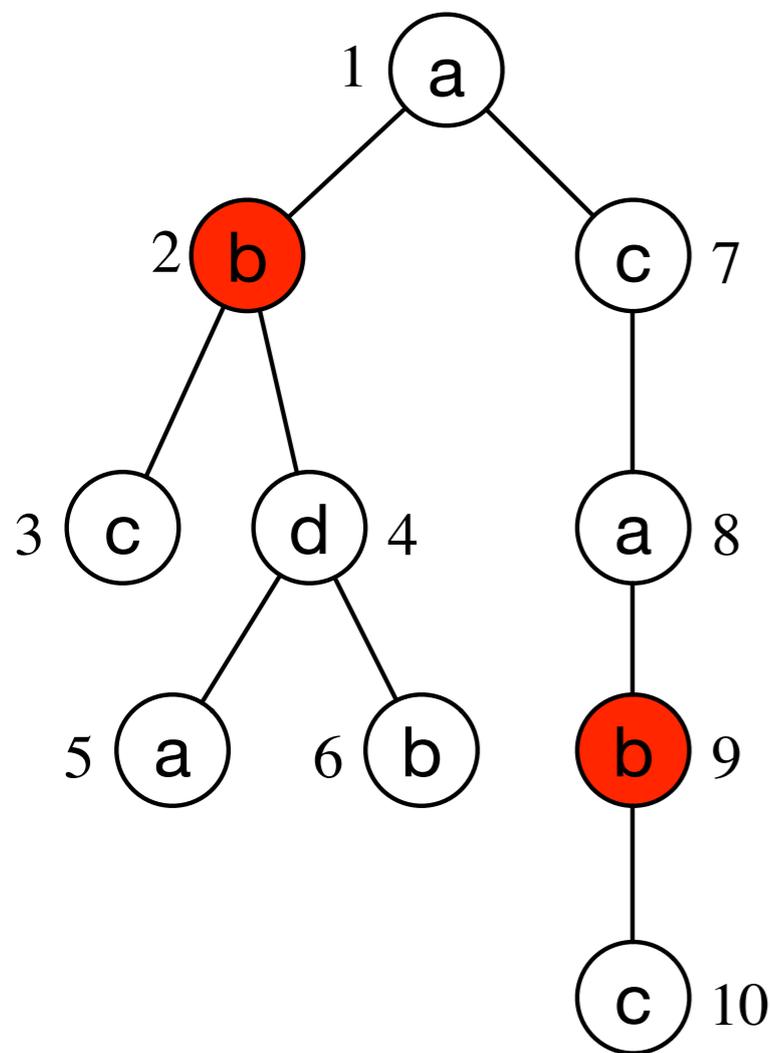
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# XPath: Bottom-up Evaluation of Core XPath

- each **set operation** and the **bookkeeping** takes  $O(|D|)$  time
- the **parse tree** is of size  $O(|Q|)$
- hence, **linear processing**:

$$O(|D| \cdot |Q|)$$

# XPath: Bottom-up Evaluation of Full XPath

- we can still have **polynomial evaluation of full XPath**  
(similar principle as Core XPath)

# XPath: Bottom-up Evaluation of Full XPath

## Context-value tables:

- each **context** in XPath can be represented using a **context-value table** (specifies situations in which a subquery should be evaluated):

$$\text{context} : \langle x, k, n \rangle$$

- determined by **preceding XPath computations** —  
bottom-up algorithm

# XPath: Bottom-up Evaluation of Full XPath

$\mathcal{E}_\uparrow : \text{Expression} \rightarrow \text{nset} \cup \text{num} \cup \text{str} \cup \text{bool},$

Expr. $E$ : Operator Signature Semantics $\mathcal{E}_\uparrow[[E]]$
location step $\chi::t : \rightarrow \text{nset}$ $\{\langle x_0, k_0, n_0, \{x \mid x_0\chi x, x \in T(t)\} \rangle \mid \langle x_0, k_0, n_0 \rangle \in \mathbf{C}\}$
location step $E[e]$ over axis $\chi$ : $\text{nset} \times \text{bool} \rightarrow \text{nset}$ $\{\langle x_0, k_0, n_0, \{x \in S \mid \langle x, \text{idx}_\chi(x, S),  S , \text{true} \rangle \in \mathcal{E}_\uparrow[[e]]\} \rangle \mid \langle x_0, k_0, n_0, S \rangle \in \mathcal{E}_\uparrow[[E]]\}$
location path $/\pi : \text{nset} \rightarrow \text{nset}$ $\mathbf{C} \times \{S \mid \exists k, n : \langle \text{root}, k, n, S \rangle \in \mathcal{E}_\uparrow[[\pi]]\}$
location path $\pi_1/\pi_2 : \text{nset} \times \text{nset} \rightarrow \text{nset}$ $\{\langle x, k, n, z \rangle \mid 1 \leq k \leq n \leq  \text{dom} ,$ $\langle x, k_1, n_1, Y \rangle \in \mathcal{E}_\uparrow[[\pi_1]],$ $\bigcup_{y \in Y} \langle y, k_2, n_2, z \rangle \in \mathcal{E}_\uparrow[[\pi_2]]\}$
location path $\pi_1 \mid \pi_2 : \text{nset} \times \text{nset} \rightarrow \text{nset}$ $\mathcal{E}_\uparrow[[\pi_1]] \cup \mathcal{E}_\uparrow[[\pi_2]]$
position() : $\rightarrow \text{num}$ $\{\langle x, k, n, k \rangle \mid \langle x, k, n \rangle \in \mathbf{C}\}$
last() : $\rightarrow \text{num}$ $\{\langle x, k, n, n \rangle \mid \langle x, k, n \rangle \in \mathbf{C}\}$

# XPath: Bottom-up Evaluation of Full XPath

$\mathcal{E}_\uparrow : \text{Expression} \rightarrow \text{nset} \cup \text{num} \cup \text{str} \cup \text{bool},$

$$\mathcal{E}_\uparrow[\mathit{Op}(e_1, \dots, e_m)] := \{ \langle \vec{c}, \mathcal{F}[\mathit{Op}](v_1, \dots, v_m) \rangle \mid \vec{c} \in \mathbf{C}, \langle \vec{c}, v_1 \rangle \in \mathcal{E}_\uparrow[e_1], \dots, \langle \vec{c}, v_m \rangle \in \mathcal{E}_\uparrow[e_m] \}$$

# XPath: Bottom-up Evaluation of Full XPath

## Context-Value Principle (CVT):

- the size of each of the context-value tables is polynomial
- computing each combination step of the expression is polynomial
- hence, the computation is polynomial

# XPath: Bottom-up Evaluation of Full XPath

**GOTTLOB EXAMPLE SLIDES**

# XPath: Bottom-up Evaluation of Full XPath

## Space Complexity:

- $O(|Q|)$  relations are created,
- nset are bounded by  $O(|D|^4)$ , bool are bounded by  $O(|D|^3)$
- numbers and string computable in XPath are of size  $O(|D||Q|)$

$$O(|D|^4 \cdot |Q|^2)$$

# XPath: Bottom-up Evaluation of Full XPath

## Time Complexity:

- $O(|Q|)$  computations are needed (parse tree size is linear in the query size),
- $O(|D|^5|Q|)$  for each expression relation

$$O(|D|^5 \cdot |Q|^2)$$

# Useful Reading

- Gottlob, Koch, Pichler. “**Efficient Algorithms for Processing XPath Queries**”, VLDB 2002.
- Green, Gupta, Miklau, Onizuka, Suciu.  
“**Processing XML Streams with Deterministic Automata and Stream Indexes**”, ACM TODS 29(4), 2004.
- Benedikt, Koch. “**XPath Leashed**”, ACM Computing Surveys 41(1), 2009.